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STUDY OF COMPTON BROADENING DUE TO ELECTRON-PHOTON SCATTERING

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SUMMARY: We have investigated the effects of Compton broadening due to electron-photon scattering in hot stellar atmospheres. A purely electron-photon scattering media is assumed to have plane parallel geometry with an input radiation field localized on one side of the slab. The method is based on the discrete space theory of radiative transfer for the intensity of emitted radiation. The solution is developed to study the importance of scattering of radiation by free electrons in high temperature stellar atmospheres which produces a brodening and shift in spectral lines because of the Compton effect and the Doppler effect arising from mass and thermal motions of scattering electrons. It is noticed that the Comptonized spectrum depends on three parameters: the optical depth of the medium, the temperature of the thermal electrons and the viewing angle. We also showed that the Compton effect produces red shift and asymmetry in the line. These two effects increase as the optical depth increases. It is also noticed that the emergent specific intensities become completely asymmetric for higher optical depths.

Key words. Stars: atmospheres – Radiative transfer – Scattering

1. INTRODUCTION

Electron scattering appears to have considerable influence through Compton broadening on formation of spectral lines in objects such as the outer layers of hot stars, AGN's, Seyfert galaxies etc. Electron scattering contributes not only to the opacity of high temperature stellar atmospheres but also produces broadening and asymmetry in the spectral lines of such stars. The redistribution of photons in the line is due to Doppler motion of the electrons and the Compton effect in the scattering process. Many authors made attempts to solve the transfer equation for two level system accounting for noncoherent electron scattering with various approximations. In the following section, we shall briefly refere to some of the relevant papers.

1.1. Impact on spectral lines

Theoretical calculations of the profile of spectral lines broadened by scattering by free electrons is a difficult problem. Munch (1948, 1950) investigated the broadening of spectral lines due to thermal motions and has found this effect in a Wolf-Rayet stars atmospheres. However, Compton shifts have been neglected. Chandrasekhar (1948) examined the shifts due to Compton scattering but neglected the Doppler shifts and used a Taylor series expansion (to the first term). Peraiah (1990), Peraiah and Varghese (1991) extended this work to the spherically symmetric geometry by including the second derivative of a Taylor series. Dirac was the first to study Compton scattering and he found that thermal motions produced a non-Doppler blue shift larger than the Compton red shift. Therefore it would be interesting to study how Compton scattering changes the energy of a high frequency photon. Edmonds (1953) derived the spectral redistribution function to the second approximation. However, he solved the equation of transfer in plane parallel approximation using the first approximation of the redistribution function. He found that non-Doppler blue shift due to thermal motions is much larger than the Compton red shift, for an optical depth less than one, which is appropriate in a Wolf Rayet star. As this optical depth is much less than what is actually found in other objects, we would like to calculate the broadening due to Compton scattering in a high optical depth case and to include the second approximation by adding the terms introduced by Edmonds (1954).

Auer and Mihalas (1968) carried out numerical calculations of the profiles of broadened lines, using the redistribution function found by Hummer and Mihalas (1967). Zel'dovich et al (1972) considered nonlinear electron scattering effects to study the evolution of frequency and angular distribution of the radiation for a narrow line in an infinite medium. Nagirnev and Vedmich (1976) developed a method to study the formation of spectral lines under the combined effects of resonant and electron scattering. They proposed two methods to calculate the line profiles. The first method (based on two-dimensional linear integral equation for the intensity of the emitted radiation) is preferable when electron scattering is comparatively unimportant, while the second one (involves separating intensity into three parts, corresponding to the continuum, electron scattering and resonant scattering) is preferred in the opposite case.

In general, interactions between electron Doppler frequency shifts and thermally broadened spectral lines can cause observable broadening of lines in spectra of the hottest stars. Recent papers by Hillier (1991) and Hamann et al. (1992) demonstrate the effect in theoretical spectra of Wolf-Rayet stars. Scattering of free electrons has to be treated as noncoherent when the spectrum contains narrow spectral lines. In that case the width of line can be smaller than the electron Doppler width. This aspect of the problem was studied by Rybicki and Hummer (1994) who investigated the importance of noncoherent continuum scattering for the formation and broadening of spectral lines. Rangarajan et al. (1991) obtained the solutions for parametrized models with both partial and complete redistribution in the line. Some basic aspects were discussed by Peraiah and Srinivasa Rao (1993) in the context of Compton broadening due to electron scattering. Madej (1994) studied Compton scattering on white dwarf model atmospheres in detail and found that pure hydrogen models with a temperature of 10^5 K show depression in the X-ray continuum. In a later paper Madej (1998) computed the effects of Compton scattering for a model corresponding to the parameters of DA white dwarf HZ43 and noticed large differences between Compton and Thompson scattering models.

In Section 2, we present the relevant equations for Compton broadening along with the transfer equation in plane parallel geometry. In Section 3, we present some of the important steps for obtaining the solution of the transfer equation. Result and discussion are in Section 4. Conclusions and the Appendix are in the next two sections.

2. SOLUTION OF THE EQUATION OF TRANSFER WITH COMPTON BROADENING

The optical depth within the atmosphere is given by:

$$d\tau = -N_e \sigma_e dZ \tag{1}$$

where dZ is the length, N_e is the electron density, and σ_e is the Thompson cross-section given by

$$\sigma_e = \frac{8\pi}{3}\frac{e^4}{m^2 c^4} \tag{2}$$

where e is the electron charge, m is its mass and c is the velocity of light. The emission coefficient for Compton scattering by electrons in thermal motion is given by Edmonds (1954):

$$\begin{aligned} j(\nu,\tau,\mu) &= N_e\sigma_e \int d\omega' \, \frac{3}{4}(1+\cos^2\Theta) \cdot \\ &\cdot \int_0^\infty d\nu' I(\nu',\tau,\mu')\psi(\nu,\Theta,\nu') \end{aligned} \tag{3}$$

where primed quantities refer to the incident radiation, unprimed to the scattered radiation and

$$\mu = \cos\theta. \tag{4}$$

Further, $\mu_1 = \mu$ and $\mu_2 = \mu'$, ν is the frequency of the radiation, $d\omega'$ increment of the solid angle about the direction of the incident radiation whose intensity is $I(\nu',\tau,\mu')$, and the angle of scattering Θ, is given by:

$$\cos\Theta = \mu_1\mu_2 + \Big[(1-\mu_1^2)(1-\mu_2^2)\Big]^{\frac{1}{2}} \cos(\varphi - \varphi'). \tag{5}$$

The spectral distribution function for scattered radiation to the second approximation is given (Edmonds 1954) as:

$$\begin{aligned} \psi(\nu,\Theta,\nu') = \frac{mc}{4\pi}\Big[4\pi mkT(1-\cos\Theta)\Big]^{\frac{-1}{2}} \\ \frac{1}{\nu'}\left[1-\frac{3}{2}\left(\frac{\nu'-\nu}{\nu}\right)\right]\exp\left\{\frac{-mc^2}{4kT(1-\cos\Theta)}\right. \\ \left.\left[\frac{\nu'-\nu}{\nu}-\frac{h\nu'}{mc^2}(1-\cos\Theta)\right]^2\left[1+\frac{\nu'-\nu}{\nu}\right]^{-1}\right\}. \end{aligned} \tag{6}$$

Here k is Boltzmann's constant and h is Planck's constant. For convenience we shall define the following:

Let us express the frequencies in the form of Edmonds (1953)

$$
\begin{aligned}
\alpha &= \left[\frac{mc^2}{4kT}\right]^{\frac{1}{2}}\left(\frac{\nu-\nu_0}{\nu_0}\right)\left[1+\frac{\nu-\nu_0}{\nu_0}\right]^{\frac{1}{2}} \\
&= -\left[\frac{mc^2}{4kT}\right]^{\frac{1}{2}}\left(\frac{\lambda'-\lambda_0}{\lambda_0}\right)\left[1-\frac{1}{2}\left(\frac{\lambda'-\lambda_0}{\lambda_0}\right)\right] \quad (7)
\end{aligned}
$$

$$
\begin{aligned}
\alpha' &= \left[\frac{mc^2}{4kT}\right]^{\frac{1}{2}}\left(\frac{\nu'-\nu_0}{\nu_0}\right)\left[1+\frac{\nu'-\nu_0}{\nu_0}\right]^{\frac{1}{2}} \\
&= \left[\frac{mc^2}{4kT}\right]^{\frac{1}{2}}\left(\frac{\lambda'-\lambda_0}{\lambda_0}\right)\left[1+\frac{1}{2}\left(\frac{\lambda'-\lambda_0}{\lambda_0}\right)\right] \quad (8)
\end{aligned}
$$

where λ and λ_0 are the wavelengths corresponding to ν and fixed frequency ν_0 respectively. By introducing

$$x = \frac{\nu-\nu_0}{\Delta} \quad (9)$$

and

$$x' = \frac{\nu'-\nu_0}{\Delta} \quad (10)$$

Eqs. (7) and (8) for α and α' become

$$\alpha = \frac{x}{\sqrt{2}}\left[1+\sqrt{2}x\left(\frac{kT}{mc^2}\right)\right]^{\frac{1}{2}} \quad (11)$$

$$\alpha' = \frac{x'}{\sqrt{2}}\left[1+\sqrt{2}x'\left(\frac{kT}{mc^2}\right)\right]^{\frac{1}{2}}, \quad (12)$$

where Δ is Doppler width ($\Delta = \frac{\nu_0 v}{c}$, v being the mean thermal velocity), T is the temperature of the gas, and ν^s being the frequencies. $I(\alpha, r, \mu)$ is the specific intensity of the radiation at the radial point r, the radius vector, making an angle $\cos^{-1}\mu$ with the ray. N_e is the electron density, σ_e is the Thomson scattering coefficient equal to 6.625×10^{-25} cm^{-2}. The quantity $\Phi(\alpha,\mu;\alpha',\mu')$ is the scattering function given by Peraiah and Srinivasa Rao (1993). In terms of these variables, the emission coefficient $j(\alpha,\tau,\mu)$ is given by

$$
\begin{aligned}
&j(\alpha,\tau,\mu) = N_e\sigma_e \int \frac{d\omega'}{4\pi}\frac{\frac{3}{4}(1+\cos\Theta)}{[\pi(1-\cos\Theta)]^{\frac{1}{2}}} \\
&\int_{-\infty}^{\infty} d\alpha' I(\alpha',\tau,\mu')\left[1-\frac{3}{2}\left(\frac{4kT}{mc^2}\right)^{\frac{1}{2}}(\alpha'-\alpha)\right]\times \\
&\exp\left\{-\frac{(\alpha'-\alpha)^2}{(1-\cos\Theta)}+2\left(\frac{mc^2}{4kT}\right)^{\frac{1}{2}}\frac{h\nu_0}{mc^2}(\alpha'-\alpha)\right\} \quad (13)
\end{aligned}
$$

2.1. The equations of radiative transfer

The equation of radiative transfer in plane parallel geometry can be written as:

$$\mu\frac{\partial I(\alpha,\tau,\mu)}{\partial\tau} = I(\alpha,\tau,\mu) - \frac{j(\alpha,\tau,\mu)}{N_e\sigma_e} \quad (14)$$

The second term on RHS of Eq. (14) is the source function and we shall simplify the emission coefficient j so that a simple solution of the Eq. (14) can be found. We shall perform the integration over the azimuthal angle φ (refer to Eq. (5)). The trigonometric functions involving the angle Θ are simplified and finally we obtain:

$$
\begin{aligned}
&j(\alpha,\tau,\mu) = \frac{3}{16\pi}N_e\sigma_e \int\int d\mu' d\alpha' I(\alpha',\tau,\mu')Y_s \\
&\left[1-\frac{3}{2}\left(\frac{4kT}{mc^2}\right)^{\frac{1}{2}}(\alpha'-\alpha)\right] \\
&\exp\left\{(\alpha'-\alpha)\left(\frac{mc^2}{4kT}\right)^{\frac{1}{2}}\frac{h\nu_0}{mc^2}-(\alpha'-\alpha)^2\right\}, \quad (15)
\end{aligned}
$$

where:

$$
\begin{aligned}
Y_s &= 2+3M_1+\frac{27}{8}\left(2M_1^2+M_2^2\right)+ \\
&+ \frac{91}{24}\left(2M_1^3+3M_1M_2^2\right)+ \\
&+ \frac{217}{48}\left(2M_1^4+6M_1M_2^2+\frac{3}{4}M_2^4\right), \quad (16)
\end{aligned}
$$

with:

$$M_1 = \mu_1\mu_2 \quad (17)$$

and

$$M_2 = \left[\left(1-\mu_1^2\right)\left(1-\mu_2^2\right)\right]^{\frac{1}{2}}. \quad (18)$$

Eq. (14) can now be written as:

$$\mu\frac{dI(\alpha,\tau,\mu)}{d\tau} = I(\alpha',\tau,\mu') - S(\alpha,\tau,\mu), \quad (19)$$

where the source function is:

$$
\begin{aligned}
S(\alpha,\tau,\mu) &= \int_{-\infty}^{\infty} d\alpha' \\
&\int_{-1}^{+1}\Phi(\alpha,\mu;\alpha',\mu')I(\alpha',\tau,\mu')d\mu' \quad (20)
\end{aligned}
$$

and the quantity $\Phi(\alpha,\mu;\alpha',\mu')$ is a scattering function:

$$
\begin{aligned}
&\Phi(\alpha,\mu;\alpha',\mu') = \frac{3}{16\pi}Y_s\left[1-\frac{3}{2}\left(\frac{4kT}{mc^2}\right)^{\frac{1}{2}}(\alpha'-\alpha)\right] \\
&\exp\left\{-(\alpha'-\alpha)^2-\frac{h\nu_0}{mc^2}(\alpha'-\alpha)\left(\frac{mc^2}{KT}\right)^{\frac{1}{2}}\right\}. \quad (21)
\end{aligned}
$$

Computed source functions are plotted against τ for both cases and shown in Fig. 4.

3. METHOD OF OBTAINING THE SOLUTION OF THE TRANSFER EQUATION

The solution of radiative transfer Eq. (19) is developed by using discrete space theory of radiative transfer, following the procedure given in Peraiah and Wehrse (1978) or Wehrse and Peraiah (1979). In general, the following teps are carried out for obtaining the solution of the transfer equation:
(i) We divide the medium into a number of "cells" whose thickness is less than or equal to the critical (τ_{crit}). The critical thickness is determined on the basis of physical characteristics of the medium. τ_{crit} provides the stability and uniqueness of the solution.
(ii) Now, the integration of the transfer equation is performed on the "cell", which is a radius - angle -frequency grid bounded by $[r_n, r_{n+1}] \times [\mu_{j-\frac{1}{2}}, \mu_{j+\frac{1}{2}}] \times [x_i, x_{i+1}]$, where $\mu_{j+\frac{1}{2}} = \sum_{k=1}^{j} C_k, j = 1, 2 \ldots, J$, where C_k are the weights of Gauss Legendre formulae, and for frequency discretization the discrete points x_i and weights a_i are used: $\int_{-\infty}^{+\infty} \Phi(x) f(x) dx \approx \sum_{i=-I}^{I} a_i f(x_i), \sum_{i=-I}^{I} a_i = 1.$
(iii) By using the interaction principle described in Peraiah and Grant (1973, hereafter PG73), we obtain the reflection and transmission operators over the "cell".
(iv) Finally, we combine all the cells by the star algorithm described in PG73, and obtain the radiation field.

So, now, we can write the discrete equivalent of Eq. (19):

$$\boldsymbol{M}_m\left(\boldsymbol{I}^+_{i,n+1} - \boldsymbol{I}^+_{i,n}\right) + \tau_{n+\frac{1}{2}} \boldsymbol{I}^+_{i,n+\frac{1}{2}} = = \frac{1}{2}\tau_{n+\frac{1}{2}} \left[\boldsymbol{\Phi}^{++}_{i,i',n+\frac{1}{2}} \boldsymbol{a}^{++}_{i',n+\frac{1}{2}} \boldsymbol{C}\boldsymbol{I}^+_{1',n+\frac{1}{2}} \boldsymbol{\Phi}^{+-}_{i,i',n+\frac{1}{2}} \boldsymbol{a}^{+-}_{i',n+\frac{1}{2}} \boldsymbol{C}\boldsymbol{I}^-_{1',n+\frac{1}{2}}\right] \tag{22}$$

$$\boldsymbol{M}_m\left(\boldsymbol{I}^-_{i,n+1} - \boldsymbol{I}^-_{i,n}\right) + \tau_{n+\frac{1}{2}} \boldsymbol{I}^-_{i,n+\frac{1}{2}} = = \frac{1}{2}\tau_{n+\frac{1}{2}} \left[\boldsymbol{\Phi}^{-+}_{i,i',n+\frac{1}{2}} \boldsymbol{a}^{-+}_{i',n+\frac{1}{2}} \boldsymbol{C}\boldsymbol{I}^+_{1',n+\frac{1}{2}} \boldsymbol{\Phi}^{--}_{i,i',n+\frac{1}{2}} \boldsymbol{a}^{--}_{i',n+\frac{1}{2}} \boldsymbol{C}\boldsymbol{I}^-_{1',n+\frac{1}{2}}\right] \tag{23}$$

where:

$$\boldsymbol{\Phi}^{++}_{i,i',n+\frac{1}{2}} = \Phi(\alpha_i, +\mu_1; \alpha'_i, +\mu_2); \quad \boldsymbol{\Phi}^{--}_{i,i',n+\frac{1}{2}} = \Phi(\alpha_i, -\mu_1; \alpha'_i, -\mu_2). \tag{24}$$

Similarly, $\boldsymbol{\Phi}^{-+}$,$\boldsymbol{\Phi}^{--}$ are defined as:

$$\boldsymbol{\Phi}^{+-}_{i,i',n+\frac{1}{2}} = \Phi(\alpha_i, +\mu_1; \alpha'_i, -\mu_2); \quad \boldsymbol{\Phi}^{-+}_{i,i',n+\frac{1}{2}} = \Phi(\alpha_i, -\mu_1; \alpha'_i, +\mu_2) \tag{25}$$

and:

$$\boldsymbol{S}^{\pm}_{n+\frac{1}{2}} = (\epsilon \boldsymbol{\Phi}^{\pm}_{i,i',n+\frac{1}{2}}) \mathrm{B}_{n+\frac{1}{2}} \delta'_{kk}, \tag{26}$$

where ϵ is the probability per each scattering that a photon will be destroyed by collisional de-excitation, and B is the Planck function. Furthermore:

$$\mathbf{M_m} = \mu_{jk}\delta_{jk}, \tag{27}$$

$$\mathbf{c} = c_{jk}\delta_{jk} \tag{28}$$

μ and C being the root and weight of the angle quadrature. Eqs. (22) and (23) can be combined for all the frequency points in line and discrete equations can be written as:

$$\boldsymbol{M}_m\left(\boldsymbol{I}^+_{n+1} - \boldsymbol{I}^+_{n}\right) + \tau_{n+\frac{1}{2}} \boldsymbol{I}^+_{n+\frac{1}{2}} = = \frac{1}{2}\tau_{n+\frac{1}{2}} \left[\boldsymbol{\Phi}^{++}\mathbf{W}^{++}\mathbf{I}^+ + \boldsymbol{\Phi}^{+-}\mathbf{W}^{+-}\mathbf{I}^-\right]_{n+\frac{1}{2}} \tag{29}$$

$$\boldsymbol{M}_m\left(\boldsymbol{I}^-_{n+1} - \boldsymbol{I}^-_{n}\right) + \tau_{n+\frac{1}{2}} \boldsymbol{I}^-_{n+\frac{1}{2}} = = \frac{1}{2}\tau_{n+\frac{1}{2}} \left[\boldsymbol{\Phi}^{-+}\boldsymbol{W}^{-+}\boldsymbol{I}^+ + \boldsymbol{\Phi}^{--}\boldsymbol{W}^{--}\boldsymbol{I}^-\right]_{n+\frac{1}{2}}, \tag{30}$$

where:

$$\boldsymbol{I}^+_{n+\frac{1}{2}} = \Big[I_1 I_2 I_3 \ldots I_i\Big]^T_{n+\frac{1}{2}}, \tag{31}$$

and

$$\mathbf{W}_k = a_i\ c_j; \quad a_i = \frac{A_i \boldsymbol{\Phi}_i}{\sum_{1'=l}^{l} A_i \boldsymbol{\Phi}_{i'}}, \tag{32}$$

$$\mathbf{k} = j + (i-1)m, \quad 1 \le k \le K = ml \tag{33}$$

where m= total number of angle points, i= running index of frequency points, j= running index of the angle points, I= total number of frequency points.

$$
\boldsymbol{M} = \begin{pmatrix} \mathbf{M}_m & & & \\ & \mathbf{M}_m & & \\ & & \ddots & \\ & & & \mathbf{M}_m \end{pmatrix}
$$

and

$$
\mathbf{c} = \begin{pmatrix} \boldsymbol{c} & & & \\ & \mathbf{c} & & \\ & & \ddots & \\ & & & \mathbf{c}. \end{pmatrix} \tag{34}
$$

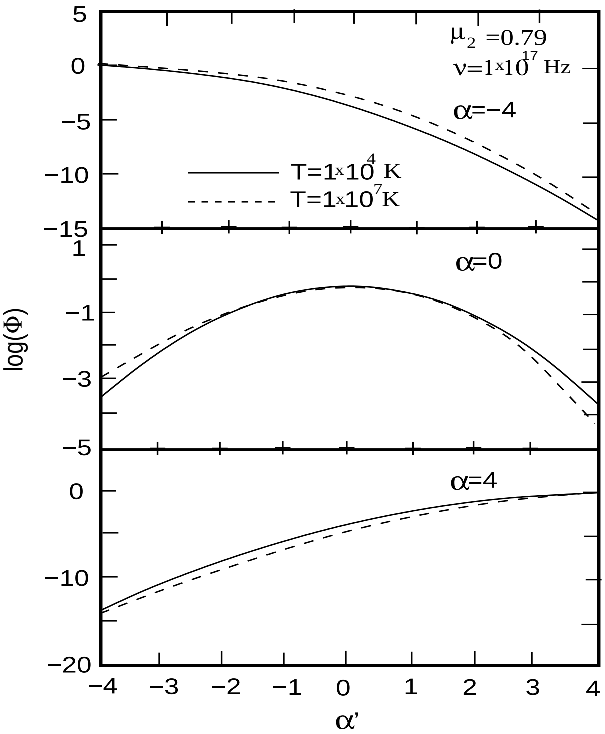

Fig. 1. *The scattering function $(\mathbf{\Phi})$ for two temperature $T = 1 \times 10^4$ K and $T = 1 \times 10^7$ K.*

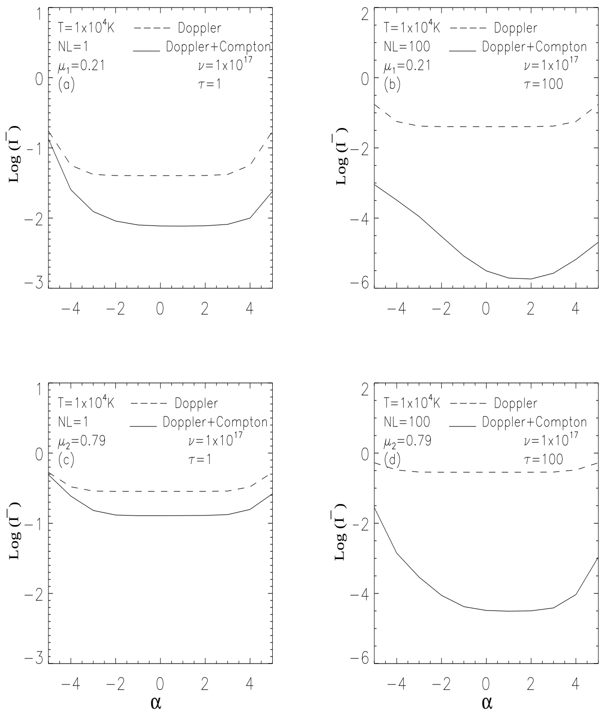

Fig. 2. *Intensity profile of I^- given for $\tau_{\min}$=1 and $\tau_{\max}$ =100 for different angles μ_1 and μ_2.*

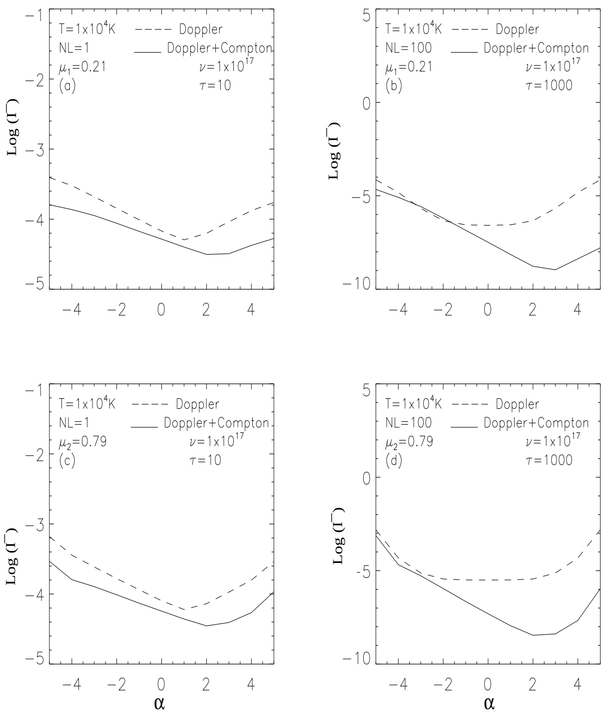

Fig. 3. *The same as Fig. 2 but for $\tau_{\min}$=10 and $\tau_{\max}$ =1000.*

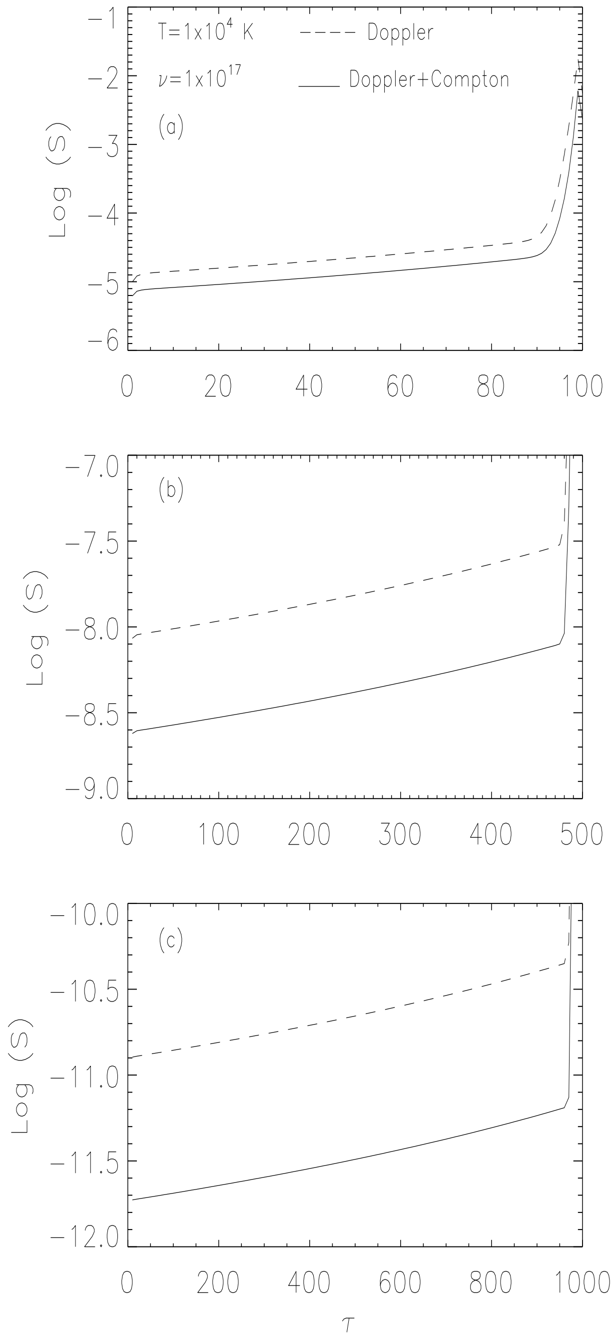

Fig. 4. *Variation of source functions against the optical depth $\tau = \tau_{\max}$=100, 500, 1000.*

Eq. (19) is integrated following the lines of PG73. The intermediate matrices **I**, **r** and **t** and internal sources Σ^s are given in Appendix and the numerical evaluation is the same as in Wehrse and Peraiah (1978). All the calculations are done in a plane parallel atmosphere.

4. RESULT AND DISCUSSION

The problem of radiative transfer in an atmosphere of free electrons is solved using the aforementioned procedure. In the solution, the account is taken of noncoherent character of electron-photon scattering due to Doppler shifts arising from thermal motions associated with kinetic temperature T which is constant throughout the atmosphere, and broadening due to Compton scattering for different optical depths. We have considered the temperature $T = 1 \times 10^4$ K for two different optical depths $\tau_{\max}$=10^2, 10^3 to study the effect of Compton broadening due to electron-photon scattering. We have also shown in Fig. 1 the variation of scattering function $\mathbf{\Phi}$ for two different temperatures $T = 1 \times 10^4$ K and $T = 1 \times 10^7$ K. Since we are using discrete space theory of radiative transfer, in general, we divide the medium into N=100 plane parallel layers of equal thickness. The layer N=100 corresponds to $\tau = \tau_{\max}$, and layer N=1 represents $\tau = 0$. We have chosen two angle points on the Gauss-Legendre quadrature (0,1) and along with normalized frequency points α_i from -4 to +4 with corresponding trapezoidal weights. We have given incident radiation at $\tau = \tau_{\max}$ and no radiation is incident at $\tau = 0$. This can be written in the following form:

$$\mathbf{I}^-\left(\mu_j, \alpha_i, \tau = \tau_{\max}\right) = 1 \;\; ; \;\; \mathbf{I}^+\left(\mu_j, \alpha_i, \tau = 0\right) = 0.$$

Fig. 1 shows the behavior of the scattering function $\mathbf{\Phi}$ with respect to the scattered frequency α' for different temperatures $T = 1 \times 10^4$, represented by solid line, and $T = 1 \times 10^7$ by dashed line. It is easily seen that the probability that a photon emitted at α=-4 with reappears at α=0 is considerably higher than that if emitted at α=+4. However, in the case where the photons are emitted either at α=-4 or α=+4, their probability of reappearing at α=+4 and α=-4, respectively, is extremely small. The photons emitted at α=0 would appear at α=±4 more strongly, although at α=-4 the probability is slightly higher than at α=+4. This happens because of the fact that the contribution comes from diffuse radiation field from the layers bounded by $\tau = 0$ and $\tau = \tau_{\max}$.

Figs. 2 and 3 show the plot of I^- against the original frequency α. The figures also show $\tau = \tau_{\min}$ at 1, 10 and $\tau = \tau_{\max}$ at $10^2, 10^3$ for two different viewing angles of μ^s respectively. We can see that the dashed curve for Doppler and the continuous curve for Doppler+Compton (Doppler+Compton, hereafter (D+C)) differ considerably.

Fig. 2 (a, b, c, d) shows Doppler effect which is symmetric with respect to the central frequency at $\alpha = 0$ for smaller and larger optical depth, $\tau = 1$ and $\tau = 100$, respectively (in some cases, in particular those shown in Fig. 3(a, c), the symmetry is not seen due to numerical instability with the method and it can be avoided by considering larger number of layers, angles and frequency points). When the Compton scattering is introduced, the asymmetry and the red shift from the central frequency are seen in the intensity profile of Figs. 2 and 3. That means that Comptomization can cause significant rise of gas temperature particularly at small optical depths, and also excess heating of the outer layers can cause interaction between photons of high thermal energies coming from hot layers and the gas which was initially cooler than the radiation. In such situation hotter photons lose energy during scattering, whereas electrons get hotter.

In the case of higher optical depth, Figs. 3 (a), 3(c) show some red shift of the line and increase at higher frequencies. This happens because of the fact that the gas temperature and photon energies are small compared to the electron mass and repeated Compton scattering. Thus, one can conclude that for $\tau > 1$, the Compton scattering produces more broadening and increases asymmetry.

Fig. 4 (a, b, c) shows the variation of source function against the optical depth τ, and reveals that larger fraction of radiation is redistributed across the medium with scattering. The source function for different optical depths $\tau_{\max}$ =100, 500, and 1000, is shown, the dashed line is for Doppler and solid line for Doppler with Compton (D+C). It can be noticed that the source functions are almost parallel to each other in the interior of the atmosphere and, as the optical depth increases, the large differences are seen in all the three cases. One can observe that the source function is increasing towards the interior of the atmosphere from $\tau = \tau_{\min}$ to $\tau = \tau_{\max}$. This is due to the fact that no radiation is incident at $\tau = 0$; when the incident radiation is given at $\tau = \tau_{\max}$, the radiation is redistributed in the atmosphere. It can be also noticed that the radiation increases as the optical depth increases. We also observe that the combined source function values (D+C) are less than the Doppler source function values.

5. CONCLUSIONS

We have investigated the Compton broadening due to electron-photon scattering in a hot stellar atmosphere in a plane parallel medium using the discrete space theory of radiative transfer and noticed that the Compton effect increases asymmetry in the line. This also increases when the optical depth increases. We would like to extend the above work to the case of a spherically symmetric atmospheres to study Compton broadening effects on spectral lines.

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APPENDIX

The transmission and reflection operators are given by:

$$\boldsymbol{t}(n+1,n) = \boldsymbol{G}^{+-}\left[\boldsymbol{\Delta}^{+}\boldsymbol{A} + \boldsymbol{g}^{+-}\boldsymbol{g}^{-+}\right] \tag{35}$$

$$\boldsymbol{t}(n,n+1) = \boldsymbol{G}^{-+}\left[\boldsymbol{A}^{-}\boldsymbol{D} + \boldsymbol{g}^{-+}\boldsymbol{g}^{+-}\right] \tag{36}$$

$$\boldsymbol{r}(n+1,n) = \boldsymbol{G}^{-+}\boldsymbol{g}^{-+}\left[\boldsymbol{I} + \boldsymbol{\Delta}^{+}\boldsymbol{A}\right] \tag{37}$$

$$\boldsymbol{r}(n,n+1) = \boldsymbol{G}^{+-}\boldsymbol{g}^{+-}\left[\boldsymbol{I} + \boldsymbol{\Delta}^{-}\boldsymbol{D}\right], \tag{38}$$

where **I** is the unit matrix, and cell operators are:

$$\Sigma^{+}_{n+\frac{1}{2}} = \boldsymbol{G}^{+-}\left[\boldsymbol{\Delta}^{+}\boldsymbol{S}^{+}_{n+\frac{1}{2}} + \boldsymbol{g}^{+-}\boldsymbol{\Delta}^{-}\boldsymbol{S}^{-}_{n+\frac{1}{2}}\right]\tau_{n+\frac{1}{2}} \tag{39}$$

$$\Sigma^{-}_{n+\frac{1}{2}} = \boldsymbol{G}^{-+}\left[\boldsymbol{\Delta}^{-}\boldsymbol{S}^{-}_{n+\frac{1}{2}} + \boldsymbol{g}^{-+}\boldsymbol{\Delta}^{+}\boldsymbol{S}^{+}_{n+\frac{1}{2}}\right]\tau_{n+\frac{1}{2}}, \quad (40)$$

where:

$$\boldsymbol{G}^{+-} = \left[\boldsymbol{I} - \boldsymbol{g}^{+-}\boldsymbol{g}^{-+}\right]^{-1} \quad (41)$$

$$\boldsymbol{G}^{-+} = \left[\boldsymbol{I} - \boldsymbol{g}^{-+}\boldsymbol{g}^{+-}\right]^{-1} \quad (42)$$

$$\boldsymbol{g}^{+-} = \frac{1}{2}\tau_{n+\frac{1}{2}}\boldsymbol{\Delta}^{+}\boldsymbol{Y}_{-} \quad (43)$$

$$\boldsymbol{g}^{-+} = \frac{1}{2}\tau_{n+\frac{1}{2}}\boldsymbol{\Delta}^{-}\boldsymbol{Y}_{+} \quad (44)$$

$$\boldsymbol{D} = \boldsymbol{M} - \tau_{n+\frac{1}{2}}\boldsymbol{Z}_{-} \quad (45)$$

$$\boldsymbol{A} = \boldsymbol{M} - \tau_{n+\frac{1}{2}}\boldsymbol{Z}_{+} \quad (46)$$

$$\boldsymbol{\Delta}^{+} = \left[\boldsymbol{M} + \frac{1}{2}\tau_{n+\frac{1}{2}}\boldsymbol{Z}_{+}\right]^{-1} \quad (47)$$

$$\boldsymbol{\Delta}^{-} = \left[\boldsymbol{M} + \frac{1}{2}\tau_{n+\frac{1}{2}}\boldsymbol{Z}_{-}\right]^{-1} \quad (48)$$

$$\boldsymbol{Z}_{+} = -\frac{1}{2}\left(\boldsymbol{\Phi}^{++}\boldsymbol{W}^{++}\right)_{n+\frac{1}{2}} \quad (49)$$

$$\boldsymbol{Z}_{-} = -\frac{1}{2}\left(\boldsymbol{\Phi}^{--}\boldsymbol{W}^{--}\right)_{n+\frac{1}{2}} \quad (50)$$

$$\boldsymbol{Y}_{+} = -\frac{1}{2}\left(\boldsymbol{\Phi}^{-+}\boldsymbol{W}\right)_{n+\frac{1}{2}} \quad (51)$$

$$\boldsymbol{Y}_{-} = -\frac{1}{2}\left(\boldsymbol{\Phi}^{+-}\boldsymbol{W}^{+-}\right)_{n+\frac{1}{2}}. \quad (52)$$

ИСТРАЖИВАЊЕ КОМПТОНОВОГ ШИРЕЊА УСЛЕД РАСЕЈАЊА ФОТОНА НА СЛОБОДНИМ ЕЛЕКТРОНИМА

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Оригинални научни рад

У овом раду бавили смо се испитивањем ефеката Комптоновог ширења услед расејања фотона на слободним електронима у атомосферама врелих звезда. Претпостављено је да је геометрија средине у којој се дешава расејање планпаралелна и да упадно зрачење долази са једне стране паралелних слојева. Интензитет емитованог зрачења је добијен применом метода који се заснива на дискретизованој теорији преноса зрачења. Изведена је и решена једначина преноса у случају расејања на слободним електронима у звезданим атмосферама на високим температурама и анализирано добијено решење како би се утврдио значај Комптоновог и Доплеровог ефекта који доводе до ширења и помака спектралних линија. Примећено је да резултујући "Комптоновски" спектар зависи од три параметра: оптичке дубине средине, температуре термалних електрона и угла посматрања. Такође је показано да Комптонов ефекат изазива црвени помак као и асиметрију линија. Ова два ефекта постају значајнији како се повећава оптичка дубина средине. За велике оптичке дубине излазни интензитет зрачења постаје потпуно асиметричан.