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A SIMPLE, APPROXIMATE METHOD FOR ANALYSIS OF KERR-NEWMAN BLACK HOLE DYNAMICS AND THERMODYNAMICS

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SUMMARY: In this work we present a simple approximate method for analysis of the basic dynamical and thermodynamical characteristics of Kerr-Newman black hole. Instead of the complete dynamics of the black hole self-interaction, we consider only the stable (stationary) dynamical situations determined by condition that the black hole (outer) horizon "circumference" holds the integer number of the reduced Compton wave lengths corresponding to mass spectrum of a small quantum system (representing the quantum of the black hole self-interaction). Then, we show that Kerr-Newman black hole entropy represents simply the ratio of the sum of static part and rotation part of the mass of black hole on one hand, and the ground mass of small quantum system on the other hand. Also we show that Kerr-Newman black hole temperature represents the negative value of the classical potential energy of gravitational interaction between a part of black hole with reduced mass and a small quantum system in the ground mass quantum state. Finally, we suggest a bosonic great canonical distribution of the statistical ensemble of given small quantum systems in the thermodynamical equilibrium with (macroscopic) black hole as thermal reservoir. We suggest that, practically, only the ground mass quantum state is significantly degenerate while all the other, excited mass quantum states, are non-degenerate. Kerr-Newman black hole entropy is practically equivalent to the ground mass quantum state degeneration. Given statistical distribution admits a rough (qualitative) but simple modeling of Hawking radiation of the black hole too.

Key words. Black hole physics

1. INTRODUCTION

In this work, generalizing our previous results on Schwarzschild and Kerr-Newman black holes (Pankovic et al. 2008abc), we present a simple, approximate method for analysis of the basic dynamical and thermodynamical characteristics (Bekenstein-Hawking entropy and Hawking temperature) of Kerr-Newman black hole. Instead of the complete dynamics of Kerr-Newman black hole self-interaction, we shall consider only the stable (stationary) dynamical situations determined by the condition that the Kerr-Newman black hole (outer) horizon "circumference" (approximately we shall formally consider that outer horizon represents a sphere) holds the integer number of the reduced Compton wave lengths corresponding to mass spectrum of the small quantum system (representing the quantum Kerr-Newman black hole self-interaction).[1] Then, we show that the Kerr-Newman black hole entropy represents the ratio of the sum of static (Schwarzschild) part and rotation part of the mass of Kerr-Newman black hole on one hand, and the ground mass of small quantum system on the other hand. Also we show that black hole temperature represents the negative value of the classical potential energy of gravitational interaction between a part of black hole with reduced mass and small quantum system in the ground mass quantum state. Finally, we suggest a bosonic great canonical distribution of the statistical ensemble of given small quantum systems in the thermodynamical equilibrium with (macroscopic) Kerr-Newman black hole as thermal reservoir. We suggest that, practically, only the ground mass quantum state is significantly degenerate while all the other, excited mass quantum states are non-degenerate. Kerr-Newman black hole entropy is practically equivalent to the ground mass quantum state degeneration. Given statistical distribution admits a rough (qualitative) but simple modeling of Hawking radiation of the black hole too. In many aspects, this modeling is very close to Parikh and Wilczek modeling of Hawking radiation as tunneling (Parikh and Wilczek 1999).

2. THEORY

As it is well-known (Wald 1984), the outer "horizon" radius (of course, Kerr-Newman outer horizon does not represent exactly a sphere, but here, as well as further in this work, we shall suppose approximately that it is a sphere with corresponding radius R) of Kerr-Newman black hole is given by

$$R = M + (M^2 - a^2 - Q^2)^{\frac{1}{2}} \quad (1)$$

where M is the black hole mass, $a = \frac{J}{M}$ where J represents the black hole angular momentum, while Q is the black hole electric charge. It implies

$$M = \frac{R}{2} + \frac{1}{2}\frac{a^2}{R} + \frac{1}{2}\frac{Q^2}{R} = M_s + M_r + M_c. \quad (2)$$

The first part of M, $M_s = \frac{R}{2}$, can be considered as an effective black hole mass corresponding to a fictitious Schwarzschild black hole with horizon radius R. In fact, M_s can be regarded as the mass corresponding to the static part of the gravitational field of Kerr-Newman black hole.

The second part of M, $M_r = \frac{1}{2}\frac{a^2}{R}$, represents classically the mass, i.e. rotation kinetic energy corresponding to angular momentum $J = aM$ and radius R.

We can introduce the following quantities:

$$M_g = M_s + M_r = \frac{R}{2} + \frac{1}{2}\frac{a^2}{R} = \frac{R^2 + a^2}{2R} \quad (3)$$

and

$$R_g = 2M_g. \quad (4)$$

Here M_g can be regarded as an effective mass corresponding to total gravitational mass representing sum of the static and rotation mass, while R_g can be considered as horizon radius of a fictitious Schwarzschild black hole with mass M_g.

The third part of M, $M_c = \frac{1}{2}\frac{Q^2}{R}$, can be considered as an effective mass, i.e. the potential energy of electrostatic repulsion of the homogeneously charged thin shell with electrical charge Q and radius R.

Finally, we can define

$$M_{\rm red} = (M^2 - a^2 - Q^2)^{\frac{1}{2}} = M(1 - \frac{a^2 + Q^2}{M^2})^{\frac{1}{2}} \quad (5)$$

which can be considered as an effective, reduced black hole mass obtained by diminishing the real black hole mass M by means of, classically speaking, rotation ("centrifugal force") and electrostatic repulsion.

Suppose now that, for "macroscopic" (with mass many times larger than Planck mass, i.e. 1) Kerr-Newman black hole, at horizon surface there is some small (with "microscopic" masses, i.e. masses smaller than Planck mass, i.e. 1) quantum system. It can be supposed that given small quantum system at black hole horizon represents the quant of the self-interaction of black hole, or, quant of the interaction between formally separated black hole and its fields.

Further, for a "macroscopic" Kerr-Newman black hole, only stable (stationary) dynamical situations will be considered rather than the complete dynamics of its self-interaction. The stability will be determined by the following condition

$$m_n R = n\frac{1}{2\pi}, \text{ for } m_n \ll M \text{ and } n = 1, 2, ... \quad (6)$$

where m_n for $m_n \ll M$ and $n = 1, 2, ...$ represent the mass (energy) spectrum of a given small quantum system. It corresponds to

$$2\pi R = n\frac{1}{m_n} = n\lambda_{rn} \text{ for } m_n \ll M \text{ and } n = 1, 2, .. \quad (7)$$

[1]Obviously it is conceptually analogous to Bohr quantization postulate interpreted by de Broglie relation in Old, Bohr-Sommerfeld, quantum theory. Also, it can be pointed out that our formalism is not theoretically dubious, since, as it is not hard to see, it can represent an extreme simplification of a more accurate, e.g. Copeland-Lahiri (1995), string formalism for the black hole description.

where $2\pi R$ formally stands for the "circumference" of Kerr-Newman black hole outer horizon, while

$$\lambda_{rn} = \frac{1}{m_n} \tag{8}$$

represents the n-th reduced Compton wavelength of the mentioned small quantum system with mass m_n for $n = 1, 2, \ldots$. Expression (7) simply means that circumference of the Kerr-Newman black hole outer horizon contains exactly n corresponding n-th reduced Compton wave lengths of given small quantum system with mass m_n captured at black hole horizon surface, for $n = 1, 2, \ldots$. Obviously, it is essentially analogous to well-known Bohr's angular momentum quantization postulate interpreted via de Broglie relation. Moreover, in more accurate quantum mechanical analysis Bohr-de Broglie standing waves turn out in Schrödinger stationary quantum states, while our reduced Compton waves turn out in quantized small oscillations of Copeland-Lahiri circular (string) loop (Copeland and Lahiri 1995.) However, there is a principal difference with respect to Bohr's atomic model. Namely, in Bohr's atomic model different quantum numbers $n = 1, 2, \ldots$, correspond to different circular orbits (with circumferences proportional to $n^2 = 1^2, 2^2, \ldots$). Here any quantum number $n = 1, 2, \ldots$ corresponds to the same circular orbit (with circumference $2\pi R$).

According to (6) and (1) there follows

$$\begin{aligned} m_n &= n\frac{1}{2\pi R} = \\ &= n\frac{1}{2\pi(M + (M^2 - a^2 - Q^2)^{\frac{1}{2}})} \equiv nm_1 \\ &\quad \text{for} \quad m_n \ll M \quad \text{and} \quad n = 1, 2, \ldots, \end{aligned} \tag{9}$$

where

$$m_1 = \frac{1}{2\pi R} = \frac{1}{2\pi(M + (M^2 - a^2 - Q^2)^{\frac{1}{2}})} \tag{10}$$

represents the ground mass of small quantum system. Obviously, m_1 depends on M so that m_1 decreases when M increases and vice versa. For a "macroscopic" black hole, i.e. for $M \gg 1$ it follows that $m_1 \ll 1 \ll M$.

It is not difficult to see that, according to (3), the ratio of M_g, m_1 represents the well-known Bekenstein-Hawking entropy of Kerr-Newman black hole, i.e.

$$S = \frac{M_g}{m_1} = \pi(R^2 + a^2) = \frac{A}{4}, \tag{11}$$

where, according to Bekenstein assumption, $A = 4S$ represents the black hole surface area. Obviously, this represents an interesting dynamical interpretation of Kerr-Newman black hole entropy whose statistical meaning will be discussed later.

Further, according to (3)-(5), (10), let us define

$$V = -\frac{M_{\text{red}} m_1}{R_g} = -\frac{(M^2 - a^2 - Q^2)^{\frac{1}{2}}}{2\pi(R^2 + a^2)} \tag{12}$$

that can be considered as the classical potential of the gravitational interaction between an effective black hole part with mass M_{red} and a small quantum system in the ground mass state m_1 at a distance R_g.

It can be observed that

$$T = -V = \frac{M_{\text{red}} m_1}{R_g} = \frac{(M^2 - a^2 - Q^2)^{\frac{1}{2}}}{2\pi(R^2 + a^2)} \tag{13}$$

represents the well-known Hawking temperature of Kerr-Newman black hole. This represents an interesting dynamical interpretation of Kerr-Newman black hole temperature.

Thus the Kerr-Newman black hole entropy (11) and temperature (13) are interpreted phenomenologically dynamically in a simple, quasi-classical way. Also, it can be seen that for a Schwarzschild black hole, representing a special case of the Kerr-Newman black hole for $a = 0$ and $Q = 0$, we get $R = 2M$, $M_s = M_g = M_{\text{red}} = M$. It implies $S = \frac{M}{m_1}$ and $T = -V = -\frac{Mm_1}{R}$ representing intuitively a very clear and simple, "obvious", quasi-classical interpretation of Schwarzschild black hole entropy (as quotient of the black hole mass and the small quantum system ground mass) and temperature (as negative classical potential energy of the gravitational interaction between the black hole and the small quantum system in mass ground state). Vice versa, clearness and simplicity, i.e. "obviousness", of Kerr-Newman black hole entropy (11) and temperature (13) follow from fact that they represent simplest generalizations of previously interpreted Schwarzschild black hole entropy and temperature.

3. THE STATISTICAL MEANING OF KERR-NEWMAN BLACK HOLE ENTROPY

In the following, we give a deeper, statistical interpretation of Kerr-Newman black hole entropy.

Suppose that the small quantum system interacting with the (macroscopic) Kerr-Newman black hole as a thermal reservoir forms a bosonic great canonical ensemble in thermodynamical equilibrium, with mass spectrum m_n for (9), temperature T given by Eq. (13) and chemical potential μ the value of which will be determined later.

Then, according to (9), statistically averaged number, N_n of the small quantum systems with mass m_n, for $n = 1, 2, \ldots$ is given by

$$\begin{aligned} N_n &= g_n \frac{1}{\exp[\frac{m_n - \mu}{T}] - 1} = \\ &= g_n \frac{1}{\exp[\frac{nm_1 - \mu}{T}] - 1}, \\ &\quad \text{for} \quad n = 1,\ 2, \ldots, \end{aligned} \tag{14}$$

where g_n represents the degeneracy of the quantum state corresponding to m_n, for n=1, 2,....

In addition, as it is well-known too, partial entropy in the quantum state corresponding to m_n for $n = 1, 2, ...$ is given by

$$S_n = g_n \ln[1+\frac{N_n}{g_n}]+N_n \ln[1+\frac{g_n}{N_n}], \quad \text{for} \quad n = 1, 2, ... , \tag{15}$$

where g_n represents the degeneracy of the quantum state corresponding to m_n for $n = 1, 2, ...$

We shall assume here

$$g_n \simeq 1, \qquad \text{for} \qquad n \gg 1 \tag{16}$$

which, according to (14), (15) implies

$$N_n \ll 1, \qquad \text{for} \qquad n \gg 1 \tag{17}$$

and

$$S_n \simeq N_n \ll 1 \qquad \text{for} \qquad n \gg 1. \tag{18}$$

Also, we shall take here

$$g_1 = N_1. \tag{19}$$

Thus, according to (10)-(13), implies the following value of the chemical potential

$$\begin{aligned} \mu &= m_1 - T \ln 2 = m_1(1 - \frac{T}{m_1} \ln 2) = \\ &= m_1(1 - \frac{1 - \frac{M}{R}}{1 + \frac{a^2}{M^2}} \ln 2) \end{aligned} \tag{20}$$

Intuitive explanation of the assumptions (16) and (19) is very simple. The ground mass state corresponding to m_1, (energetically) closest to the (outer) horizon, maximally exposed to gravitational influence, is maximally degenerate. The highly excited quantum states corresponding to m_n for $n \gg 1$, (energetically) very distant from horizon, are not so strongly exposed to gravitational influence and are almost non-degenerate.

It can be observed that here we have a situation to some extent similar to Bose condensation. Small quantum systems maximally occupy the degenerate ground mass state, in comparison to other, practically non-degenerate mass states even if, according to (19), $\frac{N_1}{g_1}$ does not tend to infinity, but to 1.

According to (15)-(19) it follows

$$S_1 = 2 \ln 2 N_1 \simeq 1.39 N_1 \sim \ N_1 \qquad \text{for} \qquad n = 1. \tag{21}$$

It implies the following expression for usual statistically defined total entropy S

$$S = \sum_{n=1} S_n \simeq S_1 \simeq 1.39 N_1 \sim N_1 \ , \tag{22}$$

and equivalence of (22) and (11) implies

$$N_1 \simeq \frac{1}{1.39} \frac{M_g}{m_1} \simeq 0.72 \frac{M_g}{m_1} \sim \frac{M_g}{m_1}. \tag{23}$$

Then, statistically averaged total number of the small quantum systems N is given by

$$N = \sum_{n=1} N_n \simeq N_1 \simeq \frac{1}{1.39} \frac{M_g}{m_1} \simeq 0.72 \frac{M_g}{m_1} \sim \frac{M_g}{m_1} \ , \tag{24}$$

and statistically averaged black hole gravitational mass of the ensemble $< M_g >$ is given by expression

$$\begin{aligned} < M_g > &= \sum_{n=1} N_n m_n \simeq N_1 m_1 \simeq \\ &\simeq \frac{1}{1.39} M_g \simeq 0.72 M_g \sim M_g \ , \end{aligned} \tag{25}$$

which approximately corresponds to the black hole gravitational mass M_g.

In this way, we statistically founded as a satisfactory approximation all previously discussed basic thermodynamical characteristics of Kerr-Newman black hole. In other words, the suggested statistics yields results which are in a satisfactory agreement with previous thermodynamical predictions.

However, it can be noticed that the assumption (16) cannot be determined by condition (19) or some other statistical or thermodynamical expression. Therefore, this reason we shall simply suppose the following form of mass (energy) degeneracy in the general case

$$g_n = (N_1 - 1) \exp[-\frac{m_n - m_1}{T}] + 1 \quad \text{for} \quad n = 1, 2, ... \tag{26}$$

which, for $n = 1$, is equivalent to (19) and to (16) for $n \gg 1$.

4. ROUGH, QUALITATIVE DESCRIPTION OF THE BLACK HOLE RADIATION

As it was shown previously practically all the small quantum systems in the statistical ensemble occupy ground mass quantum state. Therefore, transitions (jumps) from higher to lower, quantum states, especially the ground one, cannot be the primary cause of black hole Hawking radiation in our simple model. Hence, in our model it must be taken that there are some additional, subtle dynamical processes, corresponding to the Hawking near horizon particle-antiparticle creation, which cause black hole radiation and mass decrease. On the other hand, these subtle dynamical processes must be presented in our simple, approximate model only roughly, phenomenologically. It can be effectuated in a way very close to Parikh and Wilczek (1999) modeling of Hawking radiation as tunneling, or, in further conceptual analogy, as a nuclear alpha decay.

Suppose that an arbitrary of S small quantum systems in ground mass quantum state interacts dynamically with the other $S-1$ small quantum systems in ground mass quantum state similarly as one alpha particle with other alpha particles in the alpha radioactive atomic nucleus. Then, in a way similar to the modeling of alpha decay as quantum tunneling, given interaction can be presented as the propagation of one small quantum system in the potential barrier (determined by the black hole and other small quantum systems) including possibility of the tunneling, i.e. small quantum system decay.

Suppose that given individual decay occurs statistically during some time interval Δt_1 and that energy of decayed small quantum system transforms into the black hole radiation. Then, according to Heisenberg energy-time uncertainty relation it follows

$$\Delta t_1 \simeq \frac{1}{\Delta m_1} , \qquad (27)$$

where Δm_1 represents uncertainty of the mass in the ground mass quantum state corresponding to the small quantum system mass m_1.

Suppose that the ground mass level is sharply defined, i.e. that

$$\Delta m_1 \ll m_1 \qquad (28)$$

or, for example,

$$\Delta m_1 = \frac{1}{100} m_1. \qquad (29)$$

Now, the total time interval for black hole complete evaporation can be roughly represented by the expression

$$\Delta t_{\text{tot}} \simeq S \Delta t_1. \qquad (30)$$

In the simplest case, i.e. for the Schwarzschild black hole as a special limit of the Kerr-Newman black hole, as it is not difficult to see according to (30) or, according to (10), (11), (27), (29),

$$\Delta t_{\text{tot}} \simeq 1600\pi^2 M^3 = 5027\pi M^3. \qquad (31)$$

It is very close to Hawking time interval for total evaporation of the black hole

$$\Delta t_{\text{tot}} = 5120\pi M^3. \qquad (32)$$

In this way we demonstrated that our model, very close to Parikh and Wilczek modeling of Hawking radiation as tunneling, is capable of describing roughly (qualitatively) and phenomenologically, but non-trivially, both the black hole radiation and with evaporation.

5. DISCUSSION AND CONCLUSION

In the previous sections we suggested a simple, approximate but non-trivial method for description of the basic dynamical and thermodynamical characteristics of Kerr-Newman black hole. We started from the well-known exact expressions for Kerr-Newmann black hole outer horizon. Furthermore, instead of the complete dynamics of black hole self-interaction we considered only some special, stable (stationary) dynamical situations. We assumed (postulated) that the situations mentioned are determined by the rule that the "circumference" of Kerr-Newman black hole outer horizon (formally approximately treated as a sphere) contains integer numbers of reduced Compton wave lengths corresponding to the mass spectrum of a small quantum system (quantum of black hole self-interaction). It is, of course, formally analogues to Bohr angular momentum quantization postulate interpreted by de Broglie relation in the Old, Bohr-Sommerfeld quantum theory. But, as it is not difficult to see, there is no closer connection between real physical content of Bohr and our postulate. Further we proved that ground mass of a small quantum system, introduced in corresponding, practically quasi-classical dynamical expressions, determines simply and effectively the exact black hole entropy and temperature. The black hole entropy can be considered as the quotient of the sum of the black hole static and rotation mass on one hand and the ground mass of small quantum system on the other. The black hole temperature can be regarded as the negative value of the classical potential energy of gravitational interaction between part of the black hole with reduced mass and the small quantum system in the ground mass quantum state. Using the statistical methods, i.e. a bosonic great canonical ensemble, we interpreted satisfactorily the black hole entropy as the degeneration of the small quantum system ground mass quantum state. Finally, we demonstrated that the given statistical distribution admits a rough (qualitative) but simple modeling of Hawking radiation. In many aspects it is very close to Parikh and Wilczek modeling of Hawking radiation as tunneling.

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ЈЕДНОСТАВАН АПРОКСИМАТИВНИ МЕТОД ЗА АНАЛИЗУ ДИНАМИКЕ И ТЕРМОДИНАМИКЕ КЕР-ЊУМЕНОВИХ ЦРНИХ РУПА

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Оригинални научни рад

У овом раду презентујемо један једноставан, апроксимативни метод за анализу основних динамичких и термодинамичких својстава Кер-Њуменове црне рупе. Уместо комплетне динамике самоинтеракције црне рупе разматрамо само стабилне (стационарне) динамичке ситуације одређене условом да обим "кружнице" (спољашњег) хоризонта црне рупе садржи цео број редукованих Комптонових таласних дужина кореспондентних спектру масе малог квантног система (који представља квант самоинтеракције црне рупе). Показујемо да тада ентропија Кер-Њуменове црне рупе представља једноставно количник суме стационарног и ротационог дела масе црне рупе, с једне стране, и основне масе малог квантног система, с друге стране. Такође, показујемо да температура Кер-Њуменове црне рупе представља негативну вредност класичне потенцијалне енергије гравитационе интеракције између дела црне јаме са редукованом масом и малог квантног система у основном квантном стању масе. Коначно, предлажемо бозонску велику канoничку расподелу статистичког ансамбла малог квантног система у термодинамичкој равнотежи са (макроскопском) црном рупом као топлотним резервоаром. Ми сугеришемо да је, практично, само основно квантно стање масе значајно дегенерисано, док су сва остала, побуђена квантна стања масе, недегенерисана. Ентропија Кер-Њуменове црне рупе је практично еквивалентна дегенерацији основног квантног стања масе. Дата статистичка дистрибуција дозвољава, такође, и једно грубо (квалитативно) моделирање Хокинговог зрачења.