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Original scientific paper

ON THE CONVECTIVE CORES OF MS ROTATING STARS

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SUMMARY: An analysis of the influence of the uniform stellar rotation on masses and radii of convective cores of zero age stars is presented. The study also shows the dependence of the relative mass of the convective core on the parameters in the center (temperature, pressure) and on the surface (effective temperature, luminosity) of the star.

Key words. Stars: interiors – Stars: rotation

1. INTRODUCTION

The first models of slowly rotating stars in radiative equilibrium were given by Milne (1923). Later, they were generalized by Chandrasekhar (1933), and applied to polytrophic, quasi-spherical configurations that rotate like solid bodies whose equipotential surfaces coincide with the surfaces of constant pressure.

The first detailed models for the case of fast rotation, with more realistic energy generation laws and opacity laws, as well as the surface conditions, were calculated by Faulkner et al. (1968). Sackmann (1970) calculated 60 models of stars on the Main sequence with masses in a range from 0.8 to 60 m_S (Solar masses), with angular velocities of rotation ranging from zero to the critical one, by using the double approximation method, while Kippenhahn and Thomas (1970) used their own method to consider uniformly rotating stars with 1, 5 and 15 m_S. In addition, Bodenheimer (1971) used the Self Consistent Method for a case of differential rotation in order to calculate models of massive MS stars. Sreenivasan and Wilson (1989) compared the effects of differential rotation on convective cores of massive stars with the effects of rigid rotation. Deupree (2001) presented the results of the static structure and a hydrodynamic simulation of zero age Main sequence models of rotating stars with masses between 3 and 20 m_S. The influence of rotation on the evolution and on lifetimes of different evolution phases of massive stars was investigated by Meynet and Maeder (1997, 2000, 2002).

The existing models of rotating stars, although not homogeneous in the choice of the initial assumptions, structural equations and the calculating methods, undoubtedly show unequal influence of rotation on the structure of different regions in the stellar interiors. Accordingly, one can expect influence of rotation on the mechanisms of energy transport, which is the main purpose of this study.

2. BASIC EQUATIONS

A model of a star with a given mass and uniform chemical composition, in stationary rotation around fixed axis is considered; the star is isolated in space and its electromagnetic field is neglected. In this case the resulting gravitational force $\vec{g}$ is given by the effective gravitational potential Ψ: $\vec{g} = \nabla\Psi$, and the shape of the local equipotential surfaces of rotating star depends on the model.

The structure of the zero age stars is calculated by numerical integration of the following differential equations:

$$\frac{dr_p}{dM_p} = \frac{1}{4\pi r_p^2 \rho}, \quad (1)$$

$$\frac{dp}{dM_p} = -\frac{GM_p}{4\pi r_p^4} \cdot f_p, \quad (1b)$$

$$\frac{dL_p}{dM_p} = \varepsilon_{\rm nuc}, \quad (1c)$$

$$\left(\frac{dT}{dM_p}\right)_{\rm rad} = -\frac{3\kappa}{4acT^3} \cdot \frac{L_p}{(4\pi r_p^2)^2} \cdot f_T, \quad (1d')$$

or

$$\left(\frac{dT}{dM_p}\right)_{ad} = \frac{1}{n_e+1} \cdot \frac{T}{p} \cdot \frac{dp}{dM_p}. \quad (1d'')$$

According to Kippenhahn and Thomas (1970)

$$f_p = \frac{4\pi r_p^4}{GM_p S_p} \frac{1}{\langle g^{-1}\rangle},$$
$$f_T = \left(\frac{4\pi r_p^2}{S_p}\right)^2 \left\langle g^{-1}\right\rangle^{-1} \langle g\rangle^{-1}, \quad (2)$$

$$\langle g\rangle = \frac{1}{S_p} \int\limits_{\Psi=\rm const.} g d\sigma \quad \text{and}$$
$$\left\langle g^{-1}\right\rangle = \frac{1}{S_p} \int\limits_{\Psi=\rm const.} g^{-1} d\sigma, \quad (3)$$

where S_p is the area of the equipotential surface Ψ=const., at the average distance r_p from the center of the star; the local equipotential surfaces of the model coincide with surfaces T,p,ρ = const. (ex. Tassoul 1978).

The constitutive equations of the model are:

a) $p = p_g + p_{\rm rad} = 0.8263 \cdot 10^8 \frac{\rho T}{\mu} + 2.523 \cdot 10^{-15} T^4$, (4)

with
$\mu^{-1} = X + Y/4 + Z/16 = $ const.;
X, Y, Z are mass fractions of hydrogen, helium and heavier elements respectively, where the atomic mass of oxygen is taken as the average value for all metals.

b) $\kappa = \sum \kappa_i + (\kappa_A\kappa_B + \kappa_3\kappa_4) / \sum \kappa_i$, (5)
$\sum \kappa_i = \kappa_A + \kappa_B$,

where

$$\kappa_A = \kappa_1 + \kappa_2, \;\; \kappa_B = \kappa_3 + \kappa_4;$$

the opacities κ_1, κ_2, κ_3 are calculated by interpolation formulae for pure absorption (Henyey et al. 1959, see also Angelov 1972), κ_4 is calculated by Thompson formula for scattering.

c) $\varepsilon_{pp} = f_{pp}\rho X^2 \exp(14.68 - 33.80/T_6^{1/3}) T_6^{2/3}$, (6)

$$\varepsilon_{\rm CNO} = f_{\rm CNO}\rho X N \exp(64.33 - 152.28/T_6^{1/3}) T_6^{2/3}, \quad (7)$$

where

$$f_{pp} = 1 + 0.0123 T_6^{1/3} + 0.0109 T_6^{2/3} + 0.0009 T_6,$$

$$f_{\rm CNO} = 1 + 0.0027 T_6^{1/3} + 0.00778 T_6^{2/3} + 0.000149 T_6,$$

are screen factors and $T_6 \equiv (T/10^6)$K.

The mechanism of the energy transport at local equipotential surface is defined by Schwarzschild criterion:

$$d\ln T / d\ln p \equiv \nabla = \min\left(\nabla_{\rm rad}, \nabla_{ad}\right),$$

with

$$\nabla_{\rm rad} = \frac{1}{16\pi c}\frac{\kappa p}{p_{\rm rad}}\frac{L_p}{M_p}, \quad \nabla_{ad} = \frac{1}{n_e+1},$$

where

$$n_e = \frac{3}{2}\left(2 - \frac{\beta^2}{4-3\beta}\right), \quad \beta = \frac{p_g}{p}.$$

Since the model deals with average radii of the equipotential surfaces, their shape does not much differ from the spherical, and the influence of rotation on the structure of spherically symmetric star is small:

$$\Psi(r,\Theta) = -- \frac{GM}{r} - \frac{1}{2}\Omega^2 r^2 \sin^2\Theta,$$

$$r\left(r_p,\Theta\right) = r_p\left(1 - \frac{\Omega^2}{3GM} r_p^3 P_2(\Theta)\right),$$

where Θ is spherical coordinate, and

$$P_2(\Theta) = \frac{1}{4}\left(1 + 3\cos(2\Theta)\right)$$

is second order Legendre polynomial.

Eqs. 1 are solved by the technique of Kippenhahn and Thomas, with the algorithm used by Petrovic (2001). For the analysis of the local structure at the border of the convective and radiative regions, additional algorithms were constructed in this work. They are used to determine the coordinates of the cross sections of the curves that show the values of adiabatic and radiative gradients throughout the interiors of not rotating and rotating stars with different masses. These coordinates express the values of the temperature gradients, as well as the relative radii and relative masses at the equipotential surface with $\nabla_{\rm rad}$=∇_{ad}. The numerical integration was carried out within $0 < M_P < m$ $(0 < r_P < R)$, where R and m are the average radius and the total mass of the star; for the spherically symmetrical model ($\Omega = 0$): $r = r_p$ and f_p, f_T=*1*.

3. THE RESULTS

Numerical calculation of the structure of MS (zero age) stars was carried out. The models consider stationary rotation of solid body type, for stars with given mass m, constant angular velocity Ω, and uniform chemical composition (X=0.75, Y=0.23, Z=0.02, with fraction of nitrogen N=0.3·Z). Models for 26 stellar masses, mamely $\frac{m}{m_S}$ =: 0.8, 1, 1.2, 1.4, 1.5, 1.6, 1.7, 1.8, 2, 2.25, 2.5, 2.75, 3, 4, 6, 10, 15, 20, 25, 30, 40, 50, 60, 80, 100 and 120 were formulated. For each mass m, the value of the angular velocity is varied from $\Omega = 0$ (spherically symmetrical model or "model I" hereinafter) to the critical angular velocity, $\Omega = \Omega_{\max}$ (model on the border of dynamical stability or "model II" in the following). According to the Schwarzschild criterion, the equipotentional surface, with a given radius r_p, is in stable radiative equilibrium if $\nabla_{\rm rad}<\nabla_{ad}$; otherwise, the convection is dominant mechanism of energy transport. The models do not take into account the convection on the surface of the star.

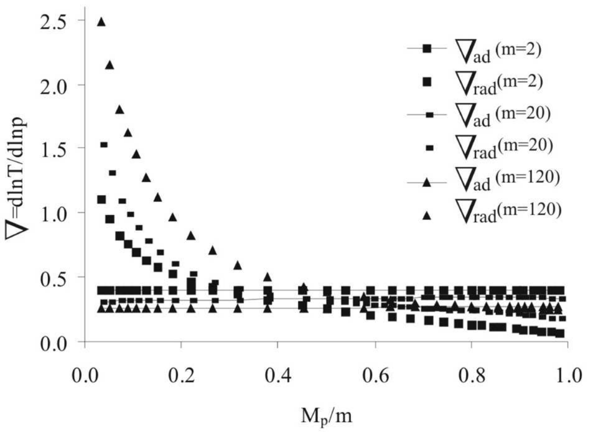

Fig. 1. *Values of adiabatic and radiative gradients throughout the interiors of stars with* $\frac{m}{m_S}$ *= 2, 20, 120, and for* $\Omega = \Omega_{max}$.

Fig. 1 shows the values of adiabatic and radiative gradients through the interiors of stars with 2, 20, 120 m_S for the rotation with critical angular velocity. It can be seen that, because of the small changes of the adiabatic gradient, the mechanism of energy transport depends mostly on the changes of values of the radiative gradient, which are faster and bigger. In deep stellar interiors, which contain the cores of the stars, the energy is transported by convection ($\nabla_{\rm rad}>\nabla_{ad}$), while the regions close to the surface of the star are in radiative equilibrium ($\nabla_{\rm rad}<\nabla_{ad}$). The equipotential surface with$\nabla_{\rm rad}=\nabla_{ad}$ defines the border of the two regions with different mechanisms of energy transport, which is also the border of the *convective core* of the star. For every stellar mass m, the influence of rotation on relative radius $r_f \equiv r/R$ of the core (R and r are the average radius of the star and its core, respectively) as well on the relative mass $q_f \equiv M_p/m$ of the core, is analyzed in this work.

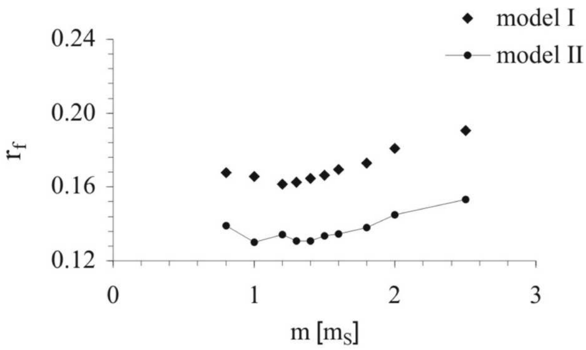

Fig. 2a. *Relative radius of convective core of star with* $m < 3\ m_S$.

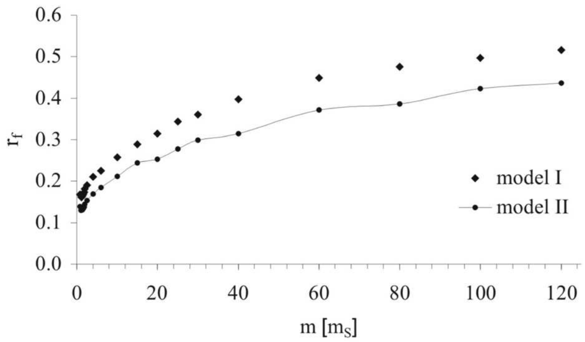

Fig. 2b. *Relative radii of convective core for various stellar masses.*

The dependence $r_f(m)$ is shown in Figs. 2a, 2b. The curve for the spherically symmetric models has minimum at 1.2m_S; for rotating models the minimum is "unstable" and lies in the range (1-1.3) m_S. For both types of models and for m>1.4m_S, the dimensions of convective cores increase as the stellar mass increases. On the other hand, rotation diminishes the dimensions of the convective core for the given mass m, and they are the smallest in critical rotation: the values of $[r_f(I)$-$r_f(II)]$ increase with the stellar mass for 1.4m_S<m<40m_S, and are almost constant for m>40m_S.

Figs. 3a, 3b show the influence of rotation on the relative radius of convective core of the star. As Ω increases, the border of the core moves fast toward the center of the star, and the dimensions of convective core are the smallest for the case of critical rotation. Table 1 shows the numerical values of $\Delta r_f=[r_f(II)$- $r_f(I)]$ and the average radius of the convective core for both types of models. $|\Delta r_f|$ is in a range from $\approx$ 0.03 to 0.08, while the relative changes are in the interval 15-22%.

Table 1. Change of r_f for models with critical rotation.

m [m_S]	Δr_f	$\Delta r_f/r_f$(I) (%)	R (I) [R_S]	R (II) [R_S]	r(I)=r_f(I)·R(I) [R_S]	r(II)=r_f(II)·R (II) [R_S]
0.8	-0.029	-17.09	1.029	1.277	0.17266	0.17763
1	-0.035	-21.46	1.115	1.390	0.18466	0.18076
1.2	-0.027	-16.94	1.176	1.478	0.18954	0.19822
1.3	-0.032	-19.68	1.213	1.509	0.19727	0.19713
1.4	-0.034	-20.59	1.244	1.557	0.20455	0.20334
1.5	-0.033	-19.83	1.262	1.567	0.20985	0.20894
1.6	-0.035	-20.76	1.284	1.601	0.21778	0.21506
1.8	-0.035	-20.20	1.343	1.669	0.23223	0.23042
2	-0.036	-19.96	1.418	1.746	0.25518	0.25283
6	-0.041	-18.16	2.702	3.268	0.60819	0.60192
10	-0.046	-17.86	3.670	4.372	0.94411	0.92367
20	-0.062	-19.78	5.440	6.687	1.71341	1.6895
40	-0.083	-20.96	7.912	9.632	3.1459	3.0274
60	-0.077	-17.24	9.830	11.521	4.4118	4.2790
100	-0.073	-14.71	12.951	14.949	6.4318	6.3319

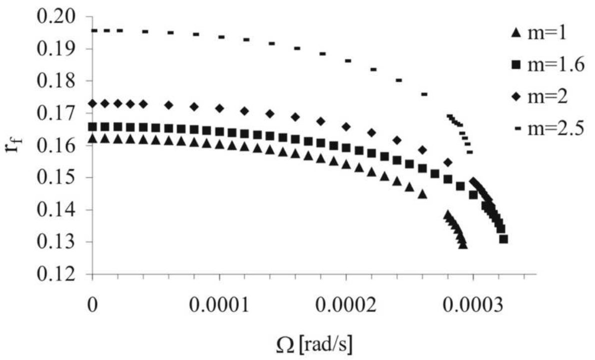

Fig. 3a. *Influence of rotation on the relative radius of convective core in stars with* $m < 3\ m_S$.

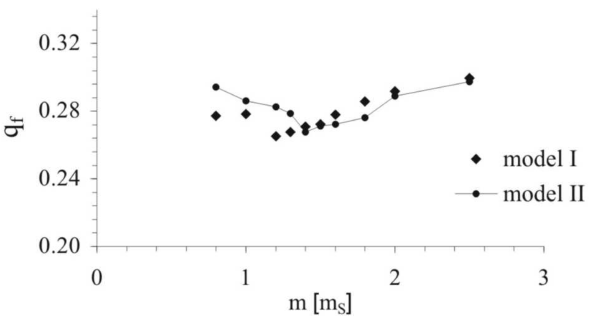

Fig. 4a. *Relative mass of convective core of stars with* $m < 3\ m_S$.

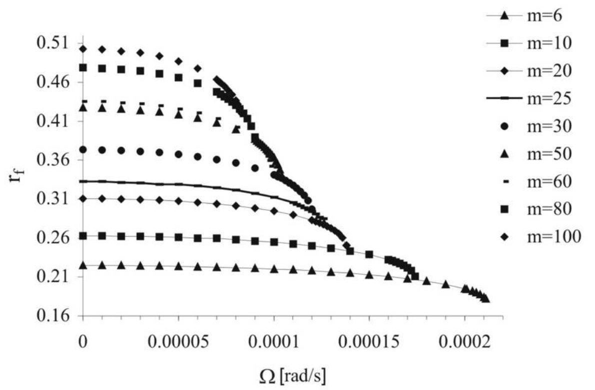

Fig. 3b. *Influence of rotation on the relative radius of convective core for massive stars.*

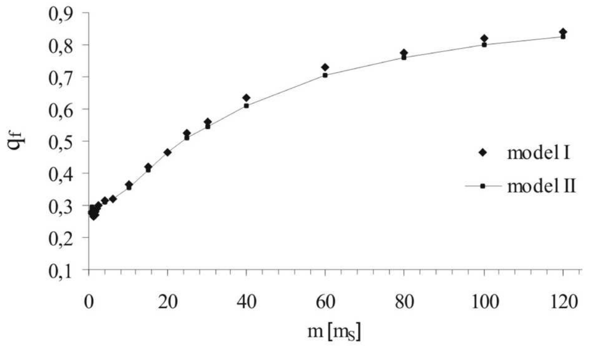

Fig. 4b. *Relative mass of convective core for various stellar masses.*

Table 2. Change of q_f for models with critical rotation.

M [m_S]	Δq_f	$\Delta q_f/q_f$(I) (%)	$M_p(I)=q_f(I)\cdot m$ [m_S]	$M_p(II)=q_f(II)\cdot m$ [m_S]
0.8	0.02	6.03	0.2219	0.2353
1	0.008	2.79	0.2782	0.2860
1.2	0.02	6.63	0.3182	0.3393
1.3	0.01	4.04	0.3481	0.3621
1.4	-0.003	-1.22	0.3793	0.3747
1.5	-0.001	-0.45	0.4086	0.4067
1.6	-0.005	-1.99	0.4447	0.4358
1.8	-0.01	-3.28	0.5146	0.4972
2	-0.0023	-0.98	0.5837	0.5778
6	-0.0004	-0.14	1.9118	1.9092
10	-0.009	-2.44	3.6467	3.5575
20	-0.004	-0.78	9.3418	9.2684
40	-0.02	-3.91	25.3621	24.3702
60	-0.02	-3.03	43.6847	42.3622
100	-0.02	-2.56	81.9432	79.8444

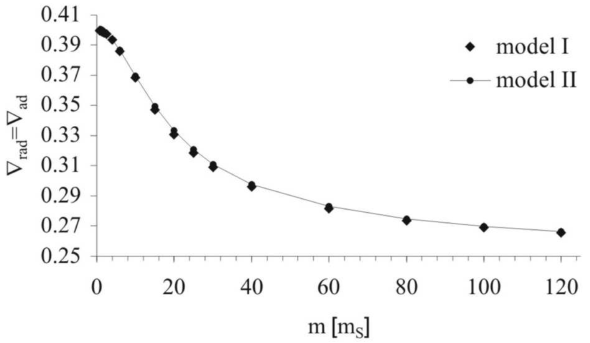

Fig. 5. *The temperature gradient* $\nabla_{rad} = \nabla_{ad}$ *for various stellar masses.*

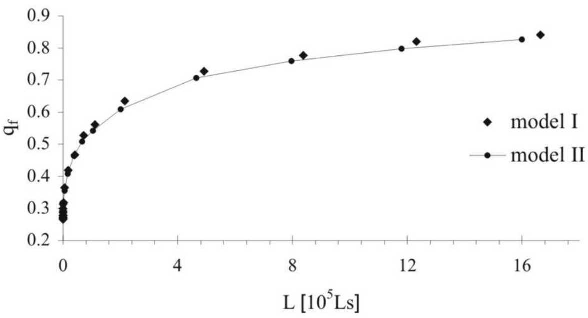

Fig. 7. *The relative mass of convective core as a function of luminosity of the star.*

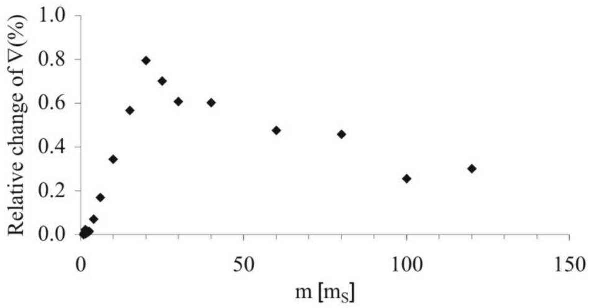

Fig. 6. *Relative change of* ∇ *for various stellar masses.*

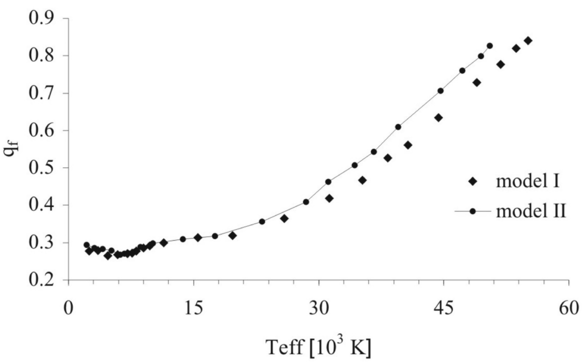

Fig. 8. *Change of the relative mass of convective core with effective temperature of star.*

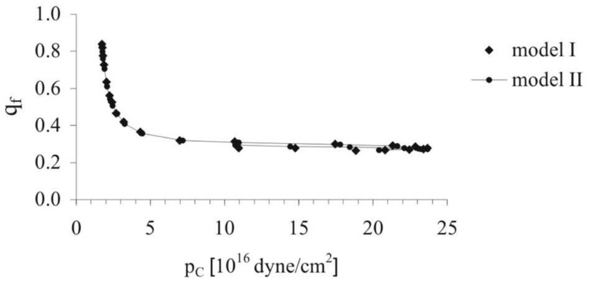

Fig. 9. *Change of* q_f *with pressure in the center of star.*

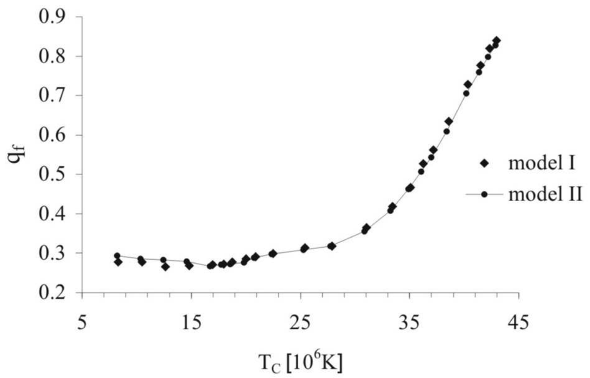

Fig. 10. *Change of* q_f *with temperature in the center of star.*

The dependence $q_f(m)$ is shown in Figs. 4a, 4b. The mass q_f has the minimum for 1.2m$_S$, for spherically symmetric models, and for 1.4m$_S$ in the case of critical rotation. For higher value of *m*, q_f increases with the stellar mass. On the other hand, the rotation decreases the mass of the convective core, the largest change being at the critical rotation, except for m<1.4m$_S$ when q_f increases monotonially with the decrease of the stellar mass, and $\Delta q_f=[q_f(II)\text{-}\ q_f(I)]>0$. As can be seen from Table 2, the relative increase of the mass of the convective core is highest for 1.2m$_S$ (6.6% compared to spherically symmetric model, or 1.8% of the mass of the whole star), while the lowest decrease is for 40m$_S$ (4% compared to spherically symmetric model, or 2.5% of the mass of the whole star).

The value of the temperature gradient at the border of the convective core decreases with the increase of the stellar mass, for both type of models (Fig. 5). On the other hand, rotation increases the value of ∇; the highest relative changes (0.8%) are for 20m$_S$ (Fig. 6).

Figs. 7, 8 show the influence of rotation on the changes of q_f with the parameters at the surface, luminosity L and effective temperature $T_{\rm eff}$. As can be noticed, q_f increases with the increase of luminosity for both rotating and non-rotating models, which is related to the fact that L and q_f increase with the increase of the stellar mass. On the other hand, the curve $q_f(T_{\rm eff})$ has a minimum at 1.2 m$_S$ for spherically symmetrical models, and at 1.4 m$_S$ for models with critical rotation. For higher $T_{\rm eff}$, $\Delta q_f>0$ as $q_f(II)$ increases faster than $q_f(I)$.

Figs. 9, 10 show the influence of rotation on the dependence of q_f on parameters in the center of star. For both type of models $q_f(p_C)$ decreases very fast for $p_C<4\times10^{16}$ dyne/cm^2. At higher central pressures, the changes of q_f are slower, and for $p_C>8\times10^{16}$ dyne/cm^2 the value of M_p turns out to be constant (with small oscillations) in about 30% of the mass of the whole star (Fig. 9). The dependence $q_f\ (\rho_C)$ is very similar to $q_f\ (p_C)$, and q_f reaches the constant value for $\rho_C>30$ g/cm^3. The negligible minimum $q_f(T_C)$, Fig. 10, at 1.2 m$_S$ for spherically symmetric models (1.4 m$_S$ for rotating models), and the fact that q_f is practically constant at low central temperatures T_C, are related to the changes in $q_f(p_C)$, $q_f(\rho_C)$: for low stellar masses (low values of *Tc*) the values of p_C, ρ_C are the highest and $q_f\approx$ const.

4. CONCLUSION

The analysis of the results shown in this study emphasizes a "critical" mass of the star, or rather, a characteristic interval (1.2-1.4 m$_S$), in which a qualitative changes in the local structure of both spherically symmetric and rotating stars occur. It is related to the fact that, in this interval of stellar masses, the CNO cycle of thermonuclear reactions of hydrogen begins to dominate pp cycle of reactions, which changes the heat regime of the star in hydrostatic equilibrium and the structure of the regions in radiative and convective equilibrium; rotation additionally changes the stellar structure. A characteristic value of 1.6 m$_S$was registered in the models of Petrovic (2001), while in the models of Sackmann (1970) it was 1.4 m$_S$. Also, by analyzing the observed values of masses and radii of components of binary systems, Angelov (1996) found a critical mass in the interval (1.2-1.4 m$_S$).

The critical value of the stellar mass also depends on the chemical composition and the opacity, which is not the topic of this work. In any case, the rotation of the stars on the Main sequence alters the distribution of mechanisms of energy transport, and thereby the structure, dimensions and masses of the regions in radiative and convective equilibrium.

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О КОНВЕКТИВНИМ ЈЕЗГРИМА РОТИРАЈУЋИХ ЗВЕЗДА ГЛАВНОГ НИЗА

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У овом раду приказан је утицај равномерне ротације на масе и радијусе конвективних језгара звезда нулте старости. Рад такође даје зависност релативне масе конвективног језгра од параметара у центру (температуре, притиска), и на површини звезде (ефективне температуре, луминозности).