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A COMPARISON OF TWO SPHERICAL MASS-DISTRIBUTION MODELS

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SUMMARY: Two spherically symmetric mass-distribution models - a special case of the generalised Schuster density law and the generalised isochrone model (both yield the same approximate density dependence on the distance in the outer parts) - are compared. It is shown that in the interval of the relative second scale length for the latter case of 0.5-0.6 the two mass distributions are almost identical. Considering to advantages, i. e. disadvantages, of the formulae describing these mass distributions this result can be of interest.

1. INTRODUCTION

There is a widely present opinion that the mass distribution within main bodies of elliptical galaxies, as well as of bulges and halos of spiral ones, can be successfully approximated with models involving spherical symmetry where in the outer parts the density decreases as r^{-4} (r distance to the centre - e. g. Dehnen, 1993). However, unlike Dehnen's paper, the present one will deal with mass distributions having a maximum at the centre. Mass distributions of such type are relatively well known. Two good examples are the generalised Schuster density law (e. g. Veltmann, 1961; Ninković, 1998) and the generalised isochrone model (Kuzmin and Veltmann, 1973). With regard that each of them has its own advantages a comparison seems reasonable and this will be the subject of the present paper.

2. THE APPROACH

As well known the density formula for the case of generalised Schuster law has the following form (e. g. Veltmann, 1961)

$$\rho(r) = \frac{\rho(0)}{[1 + (r/r_0)^2]^\beta} \ . \qquad (1)$$

Here r is, as said in Introduction, the distance to the centre of the stellar system under consideration, ρ is its density, r_0 is the scale length, whereas β is an arbitrary nonnegative number. In the present paper the subject will be the particular case - $\beta = i/2$ (i a nonnegative integer) - studied earlier by the present author (Ninković, 1998). As said in that paper this particular case has the advantage since the corresponding surface density is easily found. Besides, the density formula can be easily generalised to comprise the axial symmetry (spheroidal geometry), where the advantage concerning the calculating of the surface density remains, no matter whether the projection is done on the surface of symmetry (main surface) or on a surface perpendicular to it. With regard to what is said in Introduction a special case of the mentioned particular generalised Schuster distribution is of interest - $i = 4$ (also see Ninković, 1998). However, there is another mass distribution (density formula) also leading to the simple dependence $\rho(r) \propto r^{-4}$ in the outer parts. This is the so-called generalised isochrone potential (Kuzmin and Veltmann, 1973). Here only the expression for the density will be given

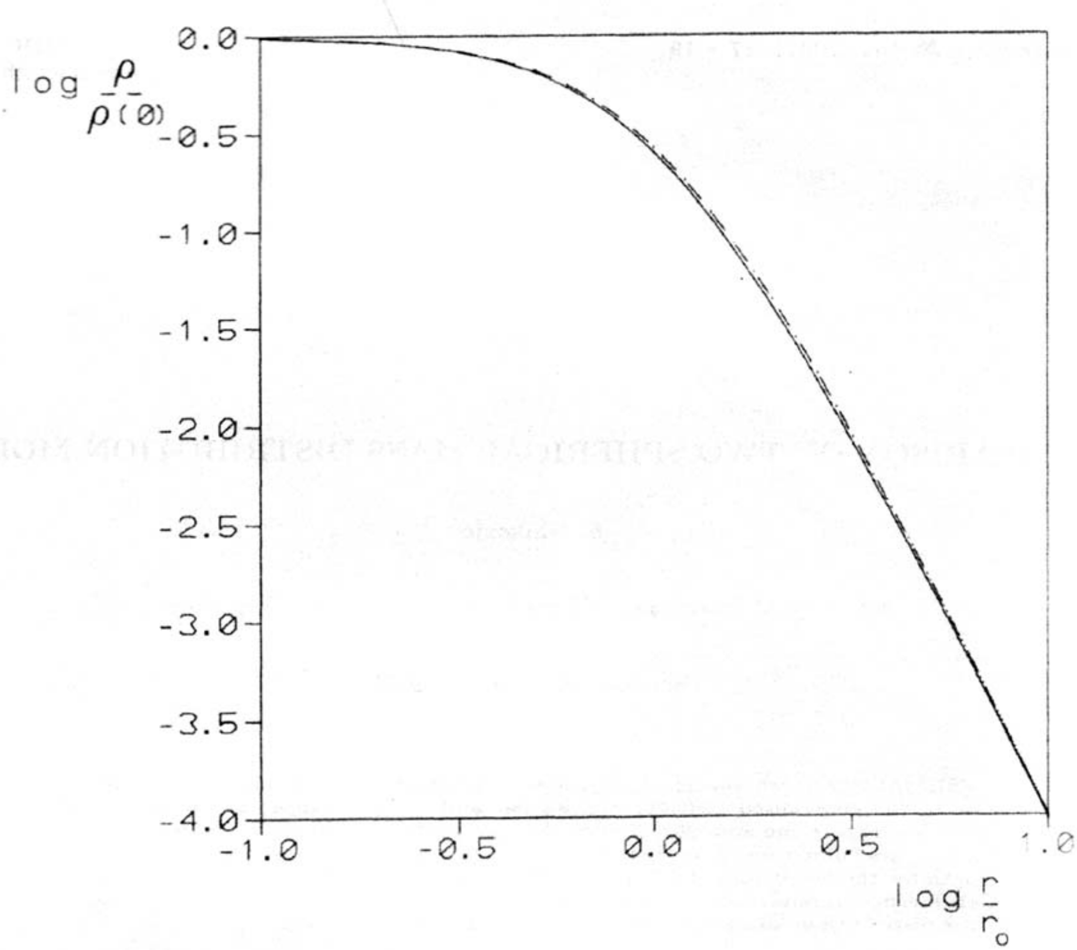

Fig. 1. *A comparison between two mass models - formula (1), $i = 4$, (solid line) and formula (2) $r_1/r_0 = 0.58$ (dashed line) - parameters $\rho(0)$ and r_0 are the same for both.*

$$\rho(r) = \frac{\mathcal{M}}{4\pi} \frac{3r_1 r_0^2 + 2r_1 r^2 + 3b^2(r^2 + r_0^2)^{1/2}}{[r_1 + (r^2 + r_0^2)^{1/2}]^3 (r^2 + r_0^2)^{3/2}} . \quad (2)$$

In this case there are two scale lengths r_1 and r_0, whereas $\mathcal{M}$ is the total mass of the stellar system.

The advantage of mass distribution (2) is that the expression for the potential is relatively simple, i. e. it can be represented with algebraic functions, unlike distribution (1), (e. g. Ninković, 1998). Due to this the potential formula can be easily generalised towards the axial symmetry. This is important for the case of calculating the orbits of individual stars. However, in the case of density formula (2) it is impossible to obtain the surface density expressed through elementary functions (for details: Kuzmin and Veltmann, 1973) and the importance of this quantity for many tasks is well known. Therefore, a comparison of the two density formulae ((1) and (2)) aimed at finding the cases of the highest similarity is of interest.

3. PROCEDURE AND RESULTS

As well known, density formula (2), though sufficiently simple for the outer parts ($\rho(r) \propto r^{-4}$), in fact covers a wide spectrum of concrete mass distributions (Kuzmin and Veltmann, 1973; Ninković, 1998). For example, in the limiting case $r_1 = 0$, r_0 different from zero, it is reduced to the classical Schuster density law and then, exceptionally, the density behaviour in the outer parts becomes $\rho(r) \propto r^{-5}$, i. e. it coincides with the particular case of (1) - $\beta = 5/2$. On the other hand, the case referred to as limiting by Kuzmin and Veltmann (1973), themselves, i. e. $r_0 = 0$, r_1 different from zero, contains a density singularity at the centre. This case was rediscovered by Hernquist (1990). Therefore, the comparison is done for the same values for $\rho(0)$ and r_0 where the ratio r_1/r_0 is taken as the comparison parameter.

Before starting this comparison it may be said that the one of the corresponding total masses yields their equality for $r_1/r_0 = 0.535$. Therefore, a good agreement may be looked for in the vicinity of this value for the comparison parameter. Indeed, within the interval of [0, 10] (the unit is r_0) the density values resulting from formulae (1)-(2) show the best agreement at r_1/r_0 equal to about 0.55. On the average the modulus of the relative density difference (with respect to density values from formula (1)) is equal to 2.8%. If the comparison is done over a larger interval, the picture is somewhat changed. For example, if it is [0, 100], then the best agreement is achieved at r_1/r_0 equal to about 0.58. The modulus of the relative density difference is on the average about 2%. This is confirmed if the comparison is done with the corresponding potentials. Therefore, mass distribution (2) when the ratio of its scale parameters r_1/r_0 is between 0.5 and 0.6 resembles mass distribution (1) (for the concrete case $i = 4$) very much. This is illustrated in Fig. 1. In view of the advantages, i. e. disadvantages, mentioned above these distributions can be mutually interchaged and in such tasks this resemblance is very important.

4. DISCUSSION AND CONCLUSIONS

The general idea of the present paper is to discuss the stellar systems where the mass distribution can be successfully described by a density function having a maximum at the centre and following approximately r^{-4} dependence in the outer parts. Two different mass distributions (formulae (1) and (2)) satisfying these conditions are analysed. It is shown that in the vicinity of a given relative value for the second scale parameter one can find an intersection of the two model families. This intersection can be used for the purpose of mutual interchanging of these families for some tasks of stellar astronomy. It should be borne in mind that family (1) allows us the surface-density expression to obtain easily, whereas in the case of (2) the potential is expressed by algebraic functions and, with regard to this, is prospective for generalising towards axial symmetry.

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ПОРЕЂЕЊЕ ДВАЈУ СФЕРНИХ МОДЕЛА РАСПОДЕЛЕ МАСЕ

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Пореде се два сферно симетрична модела расподеле масе - један посебан случај уопштеног Шустеровог закона расподеле масе и уопштени изохрони модел (оба дају исту приближну зависност густине од удаљености у спољашњим областима). Показује се да су у распону вредности релативног другог карактеристичног растојања за други случај од 0,5-0,6 ове две расподеле масе практично идентичне. С обзиром на предности, тј. недостатке, формула које описују поменуте расподеле масе овај резултат може бити интересантан.