Serb. Astron. J. № 158 (1998), 15 – 21

UDC 524.7–333
Original scientific paper

ON THE GENERALISED SCHUSTER DENSITY LAW

S. Ninković

Astronomical Observatory Volgina 7, 11160 Belgrade-74, Yugoslavia

(Received: September 21, 1998)

SUMMARY: A special case of the generalised Schuster density law for stellar systems with spherical symmetry is discussed; here the exponent in the denominator is equal to $i/2$ where i is a positive integer. Special attention is paid to the situation $2 \leq i \leq 5$ since then the mass distributions in almost all approximately spherical stellar systems and subsystems known to exist - e. g. dark coronae of galaxies, bulges and halos of spiral galaxies, as well as the systems with the classical Schuster density law - are included. With certain improvements one can also obtain more ample variants including the density continuously attaining zero at a finite radius, somewhat different descriptions of the mass distribution, as well as generalisations towards axial symmetry. It is shown among others, that a spheroid with this mass distribution ($i = 4$) yields the same total mass as the exponential disc and that the mass distribution proposed by King belongs asymptotically to the generalised Schuster density law ($i = 3$).

1. INTRODUCTION

The spherical symmetry as the most simple has been often used for the purpose of describing various stellar systems and subsystems. There are many examples: globular star clusters, dwarf ellipsoidal galaxies, bulges, halos and dark coronae of spiral galaxies, etc. In the case of observable systems various empirical formulae describing the mass distribution have been proposed: King's formula (King 1962 - formula (14)), the de Vaucouleurs formula (e. g. Binney and Tremaine 1987 - p. 21, (1-13)), the Hubble-Reynolds formula (e. g. Binney and Tremaine 1987 - p. 21, (1-14)), the Sellwood-Sanders formula (1988 - formula (1)), etc. On the other hand the dark coronae (or halos) of spiral (and not only spiral) galaxies are also thought to be nearly spherical (e. g. Trimble 1987) and for them a suitable formula has been also found (e. g. Antonov *et al.* 1975).

In the case of empirical formulae some mathematical difficulties have been frequently met, for example the impossibility of obtaining an analytical solution for the space density (e. g. Hubble-Reynolds formula and de Vaucouleurs one) or the impossibility of solving the Poisson equation for the potential analytically (King's formula). For this reason alternative formulae for the space density yielding a relatively good agreement with the given empirical ones, have been proposed. As a good example one may mention Hernquist's (1990) formula offering a satisfactory substitution for the case of the de Vaucouleurs empirical formula.

In the present paper a special class of density formulae, very simple for use from the mathematical point of view and also realistic, will be discussed.

2. THEORETICAL BASE

Let the density within a spherically symmetric stellar system be given by means of the following formula

$$\rho(r) = \frac{\rho(0)}{[1+(r/r_c)^2]^\beta}, \; r_c = \text{const} \; , \; \beta \geq 0 \; . \quad (1)$$

This formula is usually referred to as the generalised Schuster density law (e. g. Lohmann 1964). As well known, its special case - $\beta = \frac{5}{2}$ - is the classical Schuster law (also often called Plummer-Schuster law) proposed by Schuster in studying the polytrope gas spheres (e. g. Ogorodnikov 1958 - p. 460) and introduced by Plummer (1911; 1915) in stellar astronomy. It should be added that if β exceeds $\frac{3}{2}$, the radius within which formula (1) is valid may be infinite. However, in the case $\beta \leq \frac{3}{2}$, unless the system volume is finite, the resulting total mass will be infinite.

Mass distribution (1) has certain properties which make it suitable for use. Some of them were already mentioned by Veltmann (1968; 1979; 1981).

First and not mentioned by Veltmann, due to the quadratic term in the denominator density function (1) has a maximum at the centre. Moreover it the first density derivative is negative, i. e. the density, itself, decreases. In this connection the second density derivative is negative in the central parts, but at higher distances it changes the sign. Such a behaviour of the density function can be clearly seen in corresponding plots (e. g. Bouvier, 1962).

Besides, the presence of this quadratic term simplifies obtaining the surface density and the mass within a given radius r. For example, if β exceeds $\frac{3}{2}$, the expression for the surface density can be obtained analytically for any β as already mentioned by Veltmann (e. g. 1968).

However, for understandable reasons one may introduce a limitation in the present analysis, namely to assume $\beta = \frac{i}{2}$ where i is a positive integer. The case $i = 0$, though formally belongs to this limitation, is not of interest because of its triviality (constant density). Now, both the mass within a given radius and the surface density can be always obtained analytically. It should be emphasized here that Veltmann's comment concerning the divergence in the surface density for $\beta \leq \frac{1}{2}$ (Veltmann, 1979; 1981) is not acceptable. It follows from the integration of the space density over infinity in order to obtain the surface one. However, this is incorrect if $\beta \leq \frac{3}{2}$ because then, as already said above, the volume of the system must be limited in space lest the mass of the latter one be infinite.

Some particular cases concerning the value of i have been already used in the literature for the purpose of interpreting various stellar systems or classes of stellar systems as will be said below. On the other hand, the present model can be extended to comprise other kinds of symmetry as well. An example concerning the axial symmetry will be given below.

3. RESULTS

i) $i = 2$

As not difficult to see, for this particular case expression (1) becomes

$$\rho(r) = \frac{\rho(0)}{1+(r/r_c)^2} \; , \; r \leq r_l \; ;$$
$$\rho(r) = 0 \; , \; r > r_l \; . \quad (2)$$

The resulting expressions are

$$\sigma(\tilde{r}) = \frac{2\rho(0)r_c^2}{(r_c^2+\tilde{r}^2)^{1/2}} \arctan\left(\frac{r_l^2-\tilde{r}^2}{r_c^2+\tilde{r}^2}\right)^{1/2}$$

for the surface density ($\tilde{r}$ radius in projection), i. e.

$$\mathcal{M}(r) = 4\pi\rho(0)r_c^3\Big(\frac{r}{r_c} - \arctan\frac{r}{r_c}\Big)$$

for the mass within a given radius (cumulative mass). The presence of the second term in the surface-density formula (after arctan) is due to the fact that beyond $r = r_l$ the volume density is zero. If this were not the case, i. e. if the system were not limited in space, the second term, as easily seen, would yield $\frac{\pi}{2}$. However, as already said above, this case ($i = 2$) does not admit an infinite volume because it would result in an infinite total mass.

Once the cumulative-mass expression is given, it is easy to find the circular-velocity one with regard that to the circular-velocity square being equal to $G\mathcal{M}(r)/r$ (G is the universal constant of gravitaty). The circular-velocity behaviour for this case is such that it always increases, but the limiting value, when radius tends to infinity, is finite (of course, radius tending to infinity is rather a mathematical one since, as already said, radius cannot be infinite because of the total mass).

As well known, an expression of such kind has been most frequently used for the purpose of describing the mass distribution in dark coronae (or halos) of galaxies (e. g. Rohlfs and Kreitschmann 1981; Caldwell and Ostriker 1981). A disadvantage of this formula may be the discontinuity at the boundary ($r = r_l$). However, it can be removed (e. g. Ninković 1988) by adding a constant term like this

$$\rho(r) = K\Big[\frac{1}{1+(r/r_c)^2} - \frac{1}{1+(r_l/r_c)^2}\Big] \; .$$

It is clear that the new quantity K is a constant having the density dimensions. In this case the expressions for the surface density and the mass inside a given radius become somewhat altered. In both cases $\rho(0)$ should be substituted by K and then one should add $-\frac{2K}{1+(r_l/r_c)^2}(r_l^2-\tilde{r}^2)^{1/2}$ for the surface density, i. e. $-\frac{4\pi}{3}\frac{K}{1+(r_l/r_c)^2}r^3$ for the mass.

ii) $i=3$

Now expression (1) becomes

$$\rho(r)=\frac{\rho(0)}{[1+(r/r_c)^2]^{3/2}}\ ,\ r\le r_l\ ;$$

$$\rho(r)=0\ ,\ r>r_l\ . \qquad (3)$$

The corresponding expression for the surface density is

$$\sigma(\tilde{r})=\frac{2\rho(0)r_c^3}{(r_l^2+r_c^2)^{1/2}}\ \frac{(r_l^2-\tilde{r}^2)^{1/2}}{r_c^2+\tilde{r}^2}\ .$$

The expression for the cumulative mass

$$\mathcal{M}(r)=4\pi\rho(0)r_c^3\Big[\ln\Big(\frac{x+(1+x^2)^{1/2}\pm 1}{\mp x\pm(1+x^2)^{1/2}+1}\Big)-$$
$$-\frac{x}{(1+x^2)^{1/2}}\Big]\ ,\ x=\frac{r}{r_c}\ .$$

A density expression of this kind is known as the modified Hubble-Reynolds formula (e. g. Binney and Tremaine 1987 - p. 39). The one for the surface density given there, as easily seen, emanates from that given here when r_l tends to infinity. Of course, the present case ($i=3$) does not also admit an infinite system volume for the same reasons as the previous one. It has been especially used in describing the mass distribution within the bulges of (spiral) galaxies (e. g. Caldwell and Ostriker, 1981 - their general considerations).

The circular velocity increases till about 2.9 r_c (of course, if this distance is within the system) where it has a maximum, to decrease beyond. As in the previous case for $r\ge r_l$ the circular-velocity dependence on radius is that of point mass.

It is interesting to note that the mass distribution proposed by King (1962 - formulae (14), i. e. (27) and (29)) in the limiting case $\frac{r_l}{r_c}\to\infty$ takes the form of (3), i. e. that of the corresponding surface-density expression (in King's original text is used the name tidal radius - r_t - instead of limiting radius - here r_l). Thus King's density formula appears as an "asymptotical case" of the generalised Schuster law, in particular $\beta=\frac{3}{2}$.

iii) $i=4$

The form of expression (1) corresponding to this case is

$$\rho(r)=\frac{\rho(0)}{[1+(r/r_c)^2]^2}\ . \qquad (4)$$

It yields

$$\sigma(\tilde{r})=\frac{\pi}{2}\frac{\rho(0)r_c}{[1+(\tilde{r}/r_c)^2]^{3/2}}\ ;$$

$$\mathcal{M}(r)=2\pi\rho(0)r_c^3\big[\arctan\frac{r}{r_c}-\frac{r/r_c}{1+(r/r_c)^2}\big]$$

for the surface density and the cumulative mass, respectively. As for the circular velocity, it increases till about 1.8 r_c, to decrease afterwards.

This time the density can be integrated for the total mass to infinity without diverging since $\beta>\frac{3}{2}$, i. e. $i>3$. As an example of application of expression (4) may be mentioned the case of globular clusters (Kostitsyn 1922; Kolkhidashvili 1977). Such a result agrees well with the statement of Jeans (1916) according to which the density in the outer parts of globular clusters should decrease as r^{-4}. However, more recent studies of these stellar systems are in favour of King's formula which, as said above, appears as an asymptotical special case of (1) ($i=3$).

On the other hand, the circumstance that the approximate density behaviour in the outer parts is $\propto r^{-4}$ offers some comparison possibilities. Recently, independently of each other, Dehnen (1993) and Tremaine *et al.* (1994) suggested a family of models with spherical symmetry and a particular density behaviour of $\propto[r^{3-\eta}(a+r)^{1+\eta}]^{-1}$, $a=const$, $0<\eta\le 3$. This family comprises, as special cases, the formulae proposed by Jaffe (1983 - $\eta=1$) and Hernquist (1990 - $\eta=2$). It should be mentioned that, independently of Jaffe, the same formula was also found by Kuzmin *et al.* (1986). Both formulae - Jaffe's and Hernquist's - are to be applied to the elliptical galaxies (also to halos of galaxies), the latter one, as said above, approximates the de Vaucouleurs formula. If, therefore, assumed that such objects really follow a density law in which as an approximation for the outer parts appears r^{-4}, then in analogy with Dehnen (1993) and Tremaine *et al.* (1994) formula (4) can be also generalised to give rise to a family of the following form

$$\rho(r)=\frac{\rho(0)}{[1+(r/r_c)^\alpha]^\beta}\ ,\ \alpha\beta=4\ ,\ \alpha,\beta>0\ . \qquad (5)$$

Of special interest is the case when both α and β are integers and this will be treated further on. The case $\alpha=1,\beta=4$ is in common with the family found by Dehnen (1993), i. e. Tremaine *et al.* (1994).

It should be emphasized that the formulae proposed by Jaffe and Hernquist yield the same total mass for the same values of the parameters - $\rho(r_c)$ and r_c, however the former one yields higher density values within $r=r_c$ whereas the latter one yields higher density values beyond $r=r_c$. With regard to the fait that, for example, Tremaine *et al.* (1994) have already considered Jaffe's and Hernquist's formulae in the framework of the family proposed by themselves, it is interesting to consider the agreement between some of these two formulae with some

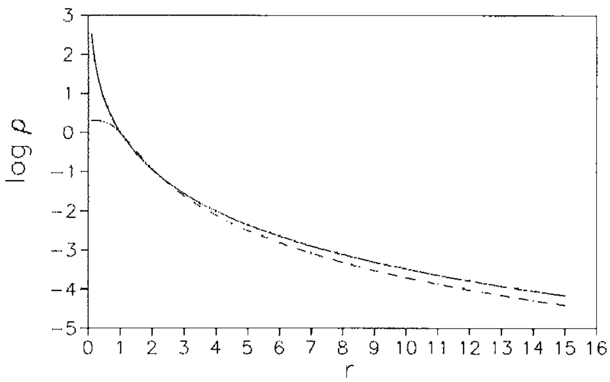

Fig. 1. *The comparison between Jaffe's density law (solid line) and the special case $\alpha = 4$, $\beta = 1$ of formula (5) (dashed line); the unit for r is r_c, that for ρ is $\rho(r_c)$.*

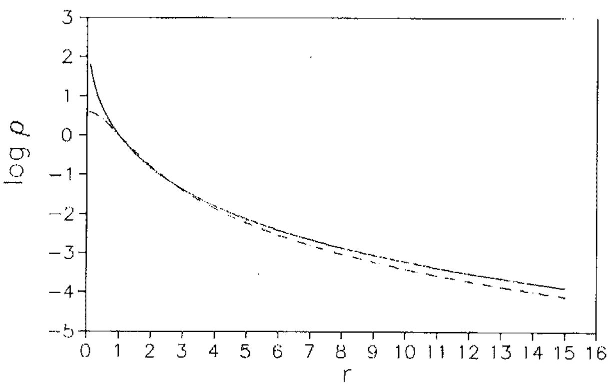

Fig. 2. *The comparison between Hernquist's density law (solid line) and the special case $\alpha = 2$, $\beta = 2$ of formula (5) (dashed line); the unit for r is r_c, that for ρ is $\rho(r_c)$.*

special case of family (5). Two specially selected cases are illustrated in Figs. 1-2.

Fig. 1 presents the comparison between the case $\alpha = 4, \beta = 1$ of family (5) and Jaffe's formula. This case gives the best agreement of all cases from family (5) (including that in common with Dehnen (1993), i. e. Tremaine *et al.* (1994)) with Jaffe's formula. As for Hernquist's one, the best agreement is achieved with $\alpha = \beta = 2$ - Fig. 2. Of course, a good agreement can be looked for only beyond $r = r_c$ because Jaffe's and Hernquist's formulae contain central cusps reaching singularities at the centre. Due to this they always yield higher total masses than the comparison formulae emanating from (5) provided that the parameters r_c and $\rho(r_c)$ are equal. Therefore, except the central regions the corresponding formulae from family (5) - $\alpha = 4, \beta = 1$ and $\alpha = \beta = 2$ - yield satisfactory agreements with Jaffe's and Hernquist's formulae, respectively. However, the question of the central regions with all its details, for example, presence of a cusp in the case of an elliptical galaxy and and to what extent it is realistic (e. g. Tremaine *et al.* 1994) is beyond the scope of the present paper.

iv) An excursion towards axial symmetry

Let formula (4) be substituted for a moment by an analogous formula where instead of r one has a new argument q and a new constant a instead of r_c. The former one is defined as

$$q = \left(R^2 + \frac{z^2}{\epsilon^2}\right)^{1/2} .$$

Here the geometry is spheroidal, i. e. the equidensit surfaces are concentric spheroids; ϵ is the axial ratio, R is the distance to the main axis (symmetry axis) and $|z|$ is the distance to the main plane (that of symmetry).

As well known, if ϵ were sufficiently small, this might correspond to the discs of spiral and lenticular galaxies (of course, assuming that their equidensit surfaces are spheroidal). It is also well known that for these discs most frequently one uses the exponential mass distribution - $\sigma(R) = \sigma(0)exp(-\alpha R)$, $\alpha = const$ (e. g. Freeman, 1970). However, the corresponding formula for the space density cannot be obtained analytically. Recently, an analytical formula for the space density fitting the numerical solution (also for the case of spheroidal geometry) was proposed by the present author (Ninković, 1997). However, the obtained formula is rather cumbersome. That formula describes the so-called reduced density ρ^*, $\rho^* = \epsilon\rho$. On the other hand, it can be noticed that (4), now modified to yield the reduced density and with new designations (q and a), and the exponential law, mentioned above, yields the same total mass, clearly provided that $\sigma(0) = \frac{\pi}{2}\rho^*(0)a$ and $a = \alpha^{-1}$. This global agreement suggests to examine the corresponding local one. The surface-density formula emanating from (4), also modified so that instead of $\tilde{r}$ and r_c one has now R and a (one deals with no projection to an arbitrary plane as in the spherical- -symmetry case but with projecting to the main one!) with $\sigma(0)$, as already mentioned, in the numerator is compared to the exponential surface-density formula. The corresponding agreement is presented in Fig. 3. As seen from this Figure, the

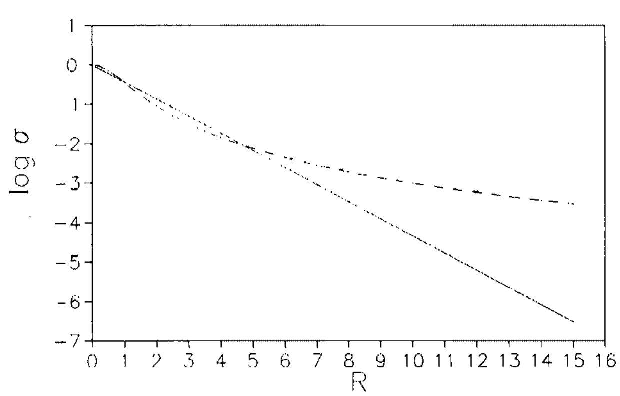

Fig. 3. *The comparison between exponential law for the surface density (solid line) and the special case $i = 4$ of the generalised Schuster law applied to spheroids (dashed line); the unit for R is a, that for σ is $\sigma(0)$.*

agreement is satisfactory, except the very external parts where it is significantly worse. However, their contribution to the total mass is practically negligible. Thus the use of (4) can make much more easy the study of discs due to its simplicity (clearly in the framework of spheroidal assumption).

It is interesting to note that as early as 1965 Schmidt in his Milky-Way model assumed a spheroidal shell surrounding the disc and within which the density decreases following q^{-4}.

v) $i = 5$

This case is the classical Schuster law. Its application in stellar astronomy has been already mentioned. However, it certaily deserves attention, above all, because it yields a very simple and very well known expression for the potential. This expression, together with the whole family emanating from it, has been rather frequently studied in the literature (e. g. Binney and Tremaine, 1987 - p. 42). The circular-velocity increase occurs till about 1.4 r_c, afterwards it decreases.

On the other hand, as is not difficult to demonstrate, within the generalised Schuster law, namely, this case yields the most simple expression for the potential. This is for the reason that unless $\beta = \frac{i}{2}$, the mass inside an arbitrary radius cannot be obtained analytically, whereas in the reverse case in the expression for $\mathcal{M}(r)$ one has for the integrand $\sin^2\varphi\cos^{i-4}\varphi$ where the argument φ is between 0 and $\arctan\frac{r}{r_c}$. The most favourable situation is, of course, when i is 5. One should say that in this case, as well as for $\beta = 0$ (form. (1)), all solutions emanating from the density formula (cumulative mass, pressure from Euler equation) are in the form of purely algebraic functions. It is curious to note that these two cases are intersection between the present family and the polytrope one.

In view of this simplicity concerning the potential corresponding to Schuster's density formula one can try a generalisation. Let the following expression be written

$$\Pi = \frac{G\mathcal{M}}{a + (r^2 + b^2)^{1/2}} \; .$$

Here Π is the potential, $\mathcal{M}$ is the total mass of the system, whereas a and b are two constants frequently called softening parameters (G see above). The corresponding expression for the density will be

$$\rho(r) = \frac{\mathcal{M}}{4\pi}\frac{3ab^2 + 2ar^2 + 3b^2(r^2 + b^2)^{1/2}}{[a + (r^2 + b^2)^{1/2}]^3(r^2 + b^2)^{3/2}}$$

Some special cases are of interest. For example $a = b$ corresponds to the isochrone potential (e. g. Binney and Tremaine 1987 - p. 38). The case $a = 0$ corresponds to the classical Schuster density law (here $i = 5$). In the literature, especially that written by Anglo- -Saxon authors, this potential is usually referred to as Plummer's one (e. g. Binney and Tremaine 1987 - p. 43). The reason, as already mentioned above, is that Plummer used the corresponding density formula for the purpose of fitting the star counts in globular clusters. Finally, if $b = 0$ and $a \neq 0$, one obtains Hernquist's density formula (Hernquist 1990) where the density is infinite at the centre unlike the cases $b \neq 0$. It is interesting to note that a further generalisation, for example

$$\Pi = \frac{G\mathcal{M}}{a + (r^n + b^n)^{1/n}} \; ,$$

where n is a positive integer, $n \geq 2$, would yield a density increasing in the central parts and, consequently, having a maximum displaced off the centre if $n > 2$.

4. DISCUSSION AND CONCLUSIONS

The generalised Schuster density law, examined in the present paper, has, undoubtedly, an ample applicability. The special case - $\beta = \frac{i}{2}$ (β exponent in denominator of (1)) being especially suitable from the mathematical point of view - can find various applications to different classes of stellar systems, i. e. subsystems. For example, the dark coronae may be considered as agreeing with $i = 2$ because in their outer parts the power- -law approximation most frequently used is r^{-2}. On the other hand, the bulges may be considered as r^{-3} systems. This is true even for some cases where formulae different from (3) are used (e. g. Caldwell and Ostriker, 1981). On the other hand, bearing in mind the usual application of King's (1962) formula to star clusters (especially globular ones) and to dwarf ellipsoidal galaxies and the fact indicated above that this formula is an asymptotical case of formula (3), one may also include these systems. It is interesting to note that for the open cluster Praesepe Bouvier (1962) found an agreement just with this particular case of the generalised Schuster density law ($i = 3$). Here one bears in mind Bouvier's interpretation of observational data (formulae (1)-(2) in that paper), not the discussion (section 5 of the same paper) where he, for some unclear reasons, assumed $i = 2$.

It is demonstrated here that the case $i = 4$ generalised towards the spheroidal geometry could be applicable to the discs of S galaxies. It could be also applicable to the halos of (spiral) galaxies and main bodies of ellipticals bearing in mind the origin of the de Vaucouleurs formula and its relationship with Hernquist's one, as well as the origin of Jaffe's formula. Here one should also mention the generalisation introduced by Dehnen (1993), i. e. by Tremaine *et al.* (1994). However, there are different halo interpretations leading to laws of the r^{-3} (e. g. van den Bergh, 1979) or $r^{-3.5}$ (e. g. Harris 1976; Zinn 1985) types. By the way, according to the present author's recent paper (Ninković 1997) an $r^{-3.5}$ distribution in the outer parts yields a better fit to the de Vaucouleurs formula than an r^{-4} one. The generalised Schuster law could be also applied to such cases comprising a new special case, namely $\beta = \frac{i}{4}$, i, as earlier, being a positive integer.

The limit concerning the special case $\beta = \frac{i}{2}$ of the generalised Schuster law introduced here ($i \leq 5$) might be thought realistic. Indeed, the corresponding potential, as already said above, acquires the most simple form - that of the point-mass potential with one softening parameter. As for stellar statistics, in favour is that the mass distribution in many stellar systems, as also mentioned above, have been successfully fitted by use of the generalised Schuster law, or similar laws, with $\beta \leq \frac{5}{2}$. However, it must be said that there are results different from these. For example, for a number of open clusters Lohmann (1964; also his papers 1972, 1976) obtained values of β exceeding $\frac{5}{2}$. The corresponding doubled values are somewhere between 4 and 8 (in other words the density decrease in the outer parts follows r^{-4} - r^{-8}). Such a sharp decrease was also noticed by King (1962) in whose models it is interpreted through the density vanishing at a finite radius. However, as demonstrated above, King's density law can be incorporated asymptotically into the Schuster generalised density law, in particular into its special case $\beta = \frac{3}{2}$.

After all, one may accept the following point of view. The generalised Schuster density law (formula (1)) has an ample application to various types of stellar systems. This application can be even generalised towards more complex symmetries as demonstrated in this paper for the particular case of axial symmetry. Its special case $\beta = \frac{i}{2}$, where i is a positive integer, is especially suitable for practical reasons. However, here limits $2 \leq i \leq 5$ seem reasonable in application to real stellar systems.

Acknowledgements –The author thanks Prof. Dr S. A. Kutuzov and Dr L. P. Osipkov for their valuable comments and advices.

This work is a part of the project "Astrometrical, Astrodynamical and Astrophysical Researches" supported by the Ministry of Science and Technology of Serbia.

REFERENCES

Antonov, V.A., Osipkov, L.P. and Chernin, A.D.: 1975, in: Dinamika i evolyutsiya zvezdnykh sistem, vyp. 4 ser. Problemy issl. vsel., ed. K.F. Ogorodnikov, AN SSSR-VAGO-GAO, Moskva-Leningrad, p. 289.

Binney, J. and Tremaine, S.: 1987, Galactic Dynamics, Princeton University Press, Princeton, New Jersey.

Bouvier, P.: 1962, *Publ. l'Observatoire Genève*, sér. A, fasc. 61, 87.

Caldwell, J.A.R. and Ostriker, J.P.: 1981, *Astrophys. J.*, **251**, 61.

Dehnen, W.: 1993, *Monthly Notices Roy. Astron. Soc.*, **265**, 250.

Freeman, K.C.: 1970, *Astrophys. J.*, **160**, 811.

Harris, W.E.: 1976, *Astron. J.*, **81**, 1095.

Hernquist, L.: 1990, *Astrophys. J.*, **356**, 359.

Jaffe, W.: 1983, *Monthly Notices Roy. Astron. Soc.*, **202**, 995.

Jeans, J.: 1916, *Monthly Notices Roy. Astron. Soc.*, **76**, 567.

King, I.: 1962, *Astron. J.*, **67**, 471.

Kolkhidashvili, M.G.: 1977, *Abastumani Byull.*, **48**, 141.

Kostitsyn, V.A.: 1922, *Trudy Glav. rossij. astron. obs.*, **1**, 28.

Kuzmin, G.G., Veltmann, Ü.-I. K. and Tenjes, P.L.: 1986, *Publ. Tartuskoj astrof. obs.*, **51**, 232.

Lohmann, W.: 1964, *Zeitschr. Astrophys.*, **60**, 43.

Lohmann, W.: 1972, *Astron. Nachr.*, **294**, 105.

Lohmann, W.: 1976, *Astrophys. Space Sci.*, **41**, 27.

Ninković, S.: 1988, *Bull. Obs. Astron. Belgrade*, **139**, 21.

Ninković, S.: 1997, *Bull. Obs. Astron. Belgrade*, **156**, 33.

Ogorodnikov, K.F.: 1958, Dinamika zvezdnykh sistem, Gos. izdat. fiz.-mat. lit., Moskva.

Plummer, H.C.: 1911, *Monthly Notices Roy. Astron. Soc.*, **71**, 460.

Plummer, H.C.: 1915, *Monthly Notices Roy. Astron. Soc.*, **76**, 107.

Rohlfs, K. and Kreitschmann, J.: 1981, *Astrophys. Space Sci.*, **79**, 289.

Schmidt, M.: 1965, In: Galactic Structure, Vol. V of Stars and Stellar Systems, eds. A. Blaauw and M. Schmidt, The University of Chicago Press, Chicago London, p. 513.

Sellwood, J.A. and Sanders, R.H.: 1988, *Monthly Notices Roy. Astron. Soc.*, **233**, 611.

Tremaine, S., Richstone, D.O., Byun, Y.-I., Dressler, A., Faber, S.M., Grillmair, C., Kormendy, J. and Lauer, T.R.: 1994, *Astron. J.*, **107**, 634.

Trimble, V.: 1987, *Ann. Rev. Astron. Astrophys.*, **25**, 425.

van den Bergh, S.: 1979, *Astron. J.*, **84**, 317.

Veltmann, Ü.-I. K.: 1968, *Itogi nauki, ser. astron.*, **4**, 7.

Veltmann, Ü.-I. K.: 1979, In: Zvezdnye skopleniya, ed. K. A. Barkhatova, Ural'skij universitet, Sverdlovsk, p. 50.

Veltmann, Ü.-I. K.: 1981, *Publ. Tartuskoj astrof. obs.*, **48**, 232.

Zinn, R.: 1985, *Astrophys. J.*, **293**, 424.

О УОПШТЕНОЈ ШУСТЕРОВОЈ ФОРМУЛИ ЗА ГУСТИНУ

С. Нинковић

Астрономска опсерваторија, Волгина 7, 11160 Београд-74, Југославија

УДК 524.7–333
Оригинални научни рад

Обрађује се један посебан случај уопштене Шустерове формуле за густину у звезданим системима са сферном симетријом; овде је изложилац у имениоцу једнак $i/2$ где је i позитиван цео број. Посебна пажња се посвећује ситуацији $2 \leq i \leq 5$ пошто су онда укључени скоро сви приближно сферни звездани системи и подсистеми за чије се постојање зна - тј. тамне короне галаксија, централни овали и халои спиралних галаксија, као и системи са класичном Шустеровом формулом за густину. Са одређеним побољшањима могу се добити и шире варијанте, на пр. густина која континуално достиже нулу на коначном растојању, нешто различити описи расподеле масе, као и уопштавања ка обртној симетрији. Између осталог, показује се да сфероид са овом расподелом масе ($i = 4$) има исту укупну масу као и експоненцијални диск и да расподела масе коју је предложио Кинг асимптотски припада уопштеној Шустеровој формули за густину ($i = 3$).